An experimental approach to the fundamental principles of hemodynamics

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THE STUDY OF BLOOD FLOW through vessels and vascular networks occupies an important place in physiology courses. Among the fundamental physical laws of hemodynamics, Bernoulli’s and Poiseuille’s equations are perhaps the most widely applied equations in the study of cardiovascular physiology, because they help us to understand many physiological phenomena of the circulatory system, either normal or pathological (3, 6, 7). Unfortunately, the fact that physiology relies heavily on the formalism of physical sciences constitutes a serious barrier for many students, who are poorly prepared to cope with it. In this context, laboratory experiments constitute an essential tool that facilitates the understanding and learning of hemodynamics.

Experiments with anesthetized animals offer the students the opportunity of confronting basic principles with observed reality. However, they have a number of drawbacks, such as the high cost and the requirement of expert time in the preparation of the animal. They pose as well ethical problems for the killing of animals for teaching purposes (8, 10). All these facts have led some schools to search for replacements for animal experiments, particularly at the undergraduate level.

Computer-based simulations are certainly an attracting alternative, and many works have been devoted to this end (1, 5, 8). Nowadays, many commercially available software programs are able to simulate the operation of the heart, lungs, and systemic circulation and the interaction among them. Thus computer simulations are usually designed to predict the behavior of complex systems that, of course, are governed by the laws of hemodynamics. However, computer simulations are of little help in demonstrating the experimental validity of these fundamental laws. In such a case, the recourse to mechanical models is a must. Mechanical models also have the advantage of involving the students in experiments that can be more easily controlled than those with anesthetized animals (2, 9). Unfortunately, mechanical models are still less frequently used in physiology teaching laboratories than computer-based models. Analog models of the vascular system where blood flow is represented by electrical current flowing in a resistance circuit have also been proposed in the past (4). However, these type of models are limited by the validity of the analogy, and the fact that the experimental and biological systems are totally different may hinder the understanding and learning of hemodynamic principles.

In this paper, we present a mechanical model that allows students to perform a number of basic experiments in hemodynamics. Contrary to other more complex mechanical models, here the emphasis is set on the study of the basic laws of hemodynamics, such as Bernoulli’s and Poiseuille’s equations, the transition from laminar to turbulent flow, or the parallel association of different vessels. These experiments have been designed so that students can directly verify the laws that govern the flow of blood and how they critically depend on parameters such as the length or diameter of vessels. Both pressure and flow rate can be directly measured by means of pressure transducers and in-line flowmeters, thus providing the student with actual and accurate experimental values. Even though the proposed mechanical model allows a precise validation of the physical laws of hemodynamics, the construction of the model is relatively simple and the cost is reasonable. The equipment required to build the system was bought in part from local suppliers (pipes, valves, etc.) and in part from RS Components, a leading supplier of industrial products (pressure transducers, flowmeters, data-acquisition systems, etc.).

PHYSICAL PRINCIPLES

In this section, we present the basic principles of fluid flow through vessels, whose experimental validation is intended. For simplicity, the elasticity of vessels will not be considered, and blood will be assumed to be an incompressible Newtonian fluid. Also, the pressure and flow pulsations during the cardiac cycle will be ignored, that is, the blood flow will be assumed to be stationary in time.

Continuity equation. Mass conservation imposes that the volume flow rate of blood (Q) entering and exiting a vessel must be identical. Because the volume flow rate is given by the product of the cross-sectional area of the vessel perpendicular to the flow (A) and the averaged velocity of blood on the section (V), the mass conservation equation can be written as...
Q = A_1V_1 = A_2V_2, \quad (1)

where subscripts 1 and 2 denote the entry and exit points in the vessel. For cylindrical vessels, \( A = \pi D^2/4 \), where \( D \) is the diameter of the vessel. Equation 1 is usually referred as the continuity equation, and a direct consequence of this equation is that when a vessel narrows (\( A_1 > A_2 \)), the averaged velocity of blood must increase (\( V_1 < V_2 \)). Of course, the opposite will happen when the cross section of the vessel increases.

When a vessel ramifies, the averaged velocity of blood in each branch is usually different because it depends on the diameter of the new branches, the lengths, etc. However, if the velocity of blood is assumed to be approximately similar in vessels of the same type (arterioles, capillaries, etc.), the continuity equation can still be applied, provided that the cross-sectional area at the exit is taken as the sum of the cross-sectional area of every vessel arising from the ramification. Because of this fact, the velocity of blood in capillaries of small diameters can be much smaller than in arterioles or arteries, because the total cross-sectional area of capillaries is substantially greater than that of arterioles and arteries.

Bernoulli’s equation. The conservation of energy of blood flow between two sections of a vessel is expressed by means of Bernoulli’s equation. In its more general form, Bernoulli’s equation can be written as

\[
p_1 + \frac{1}{2} \rho \alpha_1 V_1^2 + z_1 = p_2 + \frac{1}{2} \rho \alpha_2 V_2^2 + z_2 + \text{losses} \quad (2)
\]

where \( p \) is pressure, \( z \) is the height over a certain reference level, and \( \rho \) is blood density. The term “losses” accounts for any energy dissipation that may occur in the flow, such as the dissipation of energy due to the viscous flow of blood. The value of coefficient \( \alpha \) depends on the precise distribution of velocity on the section of the vessel. For fully developed laminar flow in a cylindrical vessel, the velocity distribution has a parabolic profile, and \( \alpha \) has a value of 2. For turbulent flow, the velocity distribution is nearly uniform, and \( \alpha \approx 1 \).

Viscous losses can usually be neglected in vessels of short length. In such cases, if the entry and exit sections of the vessel have similar heights (\( z_1 \approx z_2 \)) and cross-sectional area (\( A_1 \approx A_2 \), \( V_1 \approx V_2 \)), Bernoulli’s equation shows that pressure should also be similar. However, if the cross-sectional area of the vessel is smaller at the exit (\( A_1 > A_2 \)), the averaged velocity will be correspondingly higher (\( V_1 < V_2 \)) and pressure will decrease (\( p_1 > p_2 \)). Under certain circumstances, this pressure drop may lead to the collapse of the vessel.

Poiseuille’s law. Blood flow through organs or vascular networks is accompanied by important viscous losses, which need to be taken into account in Bernoulli’s equation. If the artery that enters and the vein that exits an organ are of similar diameters (\( V_1 \approx V_2 \)) and are located at similar heights (\( z_1 \approx z_2 \)), Bernoulli’s equation shows that there must exist a pressure drop (or perfusion pressure) associated with the viscous losses,

\[
\Delta p = p_1 - p_2 = \text{losses}.
\]

The volume flow rate through an organ under a certain perfusion pressure is determined by the resistance to the blood flow (\( R \)) according to the relation

\[
\Delta p = R \times Q. \quad (3)
\]

Therefore, any variation of the vascular resistance must be accompanied by a corresponding variation of the blood flow rate. Indeed, this is the most important mechanism of flow regulation to the organs, because perfusion pressure usually remains constant within a narrow margin.

The resistance to laminar flow in a straight vessel was determined by Jean Louis Marie Poiseuille,

\[
R = \frac{128L\eta}{\pi D^4}, \quad (4)
\]

where \( \eta \) is the dynamic viscosity of blood and \( L \) is the length of the vessel, respectively. Substitution of \( Eq. \ 4 \) into \( Eq. \ 3 \) gives the so-called Poiseuille’s law, which relates the pressure drop and volume flow rate through the vessel,

\[
\Delta p = \frac{128L\eta}{\pi D^4} \times Q. \quad (5)
\]

Poiseuille’s law shows the enormous influence that vessel diameter has on the blood flow rate that circulates through the vessel, which is the basis of many pathological and physiological phenomena (e.g., vascular tone) of the circulatory system (3, 6, 7).

In turbulent flow, the relation between pressure drop and volume flow rate is no longer linear, because the resistance to flow increases with the volume flow rate. Therefore, when the flow becomes turbulent, a much higher pressure difference is required to maintain a given flow rate. The transition from laminar to turbulent flow occurs when the Reynolds number (Re), defined as

\[
Re = \frac{\rho V D}{\eta} = \frac{4\rho Q}{\pi \eta D^2}
\]

becomes higher than a certain critical value (Re_c), typically around 2,000.

Vascular networks. In the circulatory system, vessels are interconnected, thus forming complex vascular networks. The elementary associations among vessels are in series and in parallel. A set of vessels is said to be associated in series when they are connected one after another, so that the volume flow rate through each vessel is identical. In contrast, a set of vessels is associated in parallel when they arise from the division of a main vessel at a certain point and they later meet together at a farther point. In such a case, the pressure drop in each branch will be identical because they all share the same entry and exit points. However, the volume flow rate through each branch will be, in general, different.

For vessels associated in series, the total resistance (\( R_{\text{total}} \)) to the flow of blood is simply given by

\[
R_{\text{total}} = \sum_i R_i, \quad (6)
\]

where \( i \) is the number of vessels and \( R_i \) is the resistance of every single vessel in the association. Therefore, the total resistance will always be higher than the resistance of any particular vessel in the association. Moreover, the modification of the resistance of a given vessel will only have significant
consequences on the volume flow rate if its contribution to the total resistance was important.

When vessels are associated in parallel, the total resistance of the association satisfies the relationship

\[
\frac{1}{R_{\text{total}}} = \sum \frac{1}{R_i}
\]  

(7)

Now, the total resistance will always be smaller than the resistance of any particular vessel of the association. Therefore, parallel association of vessels strongly reduces the resistance to blood flow. For this reason, capillaries, whose individual resistances are high owing to their small diameters, only represent a small fraction of the total vascular resistance of organs, because they are mainly associated in parallel.

**EXPERIMENTAL SETUP**

Figure 1 shows a schematic picture of the system used in the experiments. Basically, it consists in three independent lines where Poiseuille’s law (bottom line), Bernoulli’s equation (middle line), and the association of vessels (top line) can be studied. All three lines share a common flow control valve, and the flow is deflected to the appropriate line by means of three separated shutoff valves. In the experiments, water will play the role of blood.

Along the bottom line, the flow of water was measured by means of a miniturbine flowmeter (800 series, Titan Enterprises) in the range 0–1 l/min. The miniturbine outputs a periodic square-wave signal, the frequency of which is proportional to the flow rate through the device. In our experimental arrangement, the frequency was measured using a low-cost data-acquisition system (Pico ADC-40) connected to a personal computer, although a standard oscilloscope could also be used for this purpose. Poiseuille’s law was investigated using a 150-cm-long glass pipe with an inner diameter of 5 mm. At both ends of the glass tube, two small T junctions (143 cm distant) enabled the measurement of the pressure drop in the line, which was performed by means of a differential low-pressure transducer (0–25 mmHg, Honeywell 24PC Series). The pressure transducer outputs a low-voltage signal (<35 mV) that is linearly dependent on the pressure difference applied to the two ports of this device. In our experimental setup, the voltage (and the corresponding pressure) was measured using another data-acquisition system (Pico ADC-16),
although a standard millivoltmeter could be used as well. The transition to turbulence was visualized by injecting at the entry of the glass tube a red tracer (E-123, Amaranth). The tracer was introduced into the flow with the help of a curved needle (outer diameter: 1 mm), which was punctured in the silicone tubing that connects the miniturbine to the glass tube. The tracer flow was regulated by means of a screw clamp. The tracer reservoir was kept at a fixed height of about 1 m. This height was sufficient to ensure that the marker would be injected into the flow.

In the middle line, the flow rate was measured with the help of a low-cost rotameter (MR300 Series, Key Instruments) in the range of 50–300 ml/min. This device allows direct measurement of the flow rate at the position of the float. To investigate the relationship between the pressure drop and flow rate in a vessel contraction, a 14-cm-long glass pipe with a variable cross section was used. This glass pipe consisted of three consecutive segments. The central part of the pipe was 4 cm long with a constant inner diameter of ~2 mm. At both ends of this narrow pipe, two larger pipes, with inner diameters of 5 mm and lengths of 5 cm, were soldered. The transition from the central pipe to the outer pipes was smooth to inhibit the formation of eddies. Similar to the bottom line, two small T junctions were situated at the center of the first and second segments of the glass pipe ~5 cm apart.

Finally, in the top line, the main flow was ramified along three flexible polyurethane pipes with inner diameters of 2 mm and lengths of $L_1 = 162$ cm, $L_2 = 232$ cm, and $L_3 = 322$ cm, respectively. The pressure drop between the entry and exit points of this parallel association was measured by means of a third pressure transducer (0–260 mmHg, Honeywell 24PC Series), as shown in Fig. 1. Along each pipe, a rotameter (50–300 ml/min, MR300 Series, Key Instruments) was inserted, thus providing independent measurements of the flow rate through each branch. The presence of the rotameter introduces a small local resistance in the pipe, which is connected in series with the resistance due to flow through the pipe. This local resistance ($R_{\text{local}}$) was determined before the start of the experiment and had an approximate value of $R_{\text{local}} \approx 0.084$ mmHg·ml$^{-1}$·min$^{-1}$.

A screw clamp was also inserted in the silicone tubing, with inner diameter of 5 mm, which carries the flow to the bifurcation. As will be shown later, this clamp will be used to simulate the generation of a critical stenosis in an artery.

**RESULTS AND DISCUSSION**

**Flow through a vessel.** The first experiment is aimed at studying the relationship between pressure and flow rate through a horizontal vessel. In particular, the validity of Poiseuille’s equation will be investigated and the transition from laminar to turbulent flow will be observed.

Using the bottom line of the experimental setup, the students recorded the pressure drop between the entry and exit point of the glass pipe for increasing values of the flow rate, up to 1 l/min. Then, taking into account the dimensions of the glass pipe ($L = 143$ cm, $D = 5$ mm) and the viscosity of water ($\eta \approx 10^{-3}$ kg·m$^{-1}$·s$^{-1}$), the resistance of the vessel to the flow was calculated according to

$$R = \frac{9.322 \times 10^4 \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}}{11.65 \text{ mmHg} \cdot \text{l}^{-1} \cdot \text{min}}.$$

This value is very close to $R_e \approx 2,000$, which is considered as the lowest Re that marks the transition from laminar to turbulent flow in a circular pipe.

In the last part of this first experiment, the students had the possibility to directly observe the destabilization of the laminar flow and verify that the transition occurs precisely when Poiseuille’s law ceased to be valid. This observation was possible thanks to the injection of a red tracer into the flow.

**Narrowing of a vessel.** The study of the pressure drop when the cross section of a vessel suffers a reduction is the objective of this second experiment. The middle line of the experimental setup is used in the present case.
In this experiment, the students recorded the pressure difference that appears between the larger and smaller sections of the glass pipe (which simulates the obstruction of a blood vessel) for different values of the flow rate. The flow rate was measured with the help of the rotameter installed in the line.

To compare the experimental measurements and the theory behind this phenomenon, a number of previous considerations need to be taken into account. First, owing to the small distance between the points where the pressure is measured, the pressure drop due to the viscous flow can be neglected. This is particularly true in the laminar regime, where the resistance to the flow is smaller, as seen previously. This fact will not be certainly the case for the vessel with smaller diameter, because the short entrance length does not allow the establishment of a developed profile. Instead, the velocity profile is approximately parabolic (α ≈ 2). However, this will not be certainly the case for the vessel with smaller diameter, because the short entrance length does not allow the establishment of a developed profile. Instead, the velocity profile will be rather uniform (α ≈ 1). Taking into account these considerations, Bernoulli’s equation predicts a pressure drop between the two sections of the vessel that is not linear, but quadratic, to the flow rate, and that is given by

$$\Delta p = \frac{1}{2} \rho (\alpha_2 V_2^2 - \alpha_1 V_1^2) = \frac{8 \rho}{\pi} \left( \frac{\alpha_2}{D_2^4} - \frac{\alpha_1}{D_1^4} \right) Q^2.$$

Figure 4 shows the comparison between the theory and experiment for $\alpha_1 = 2$ and $\alpha_2 = 1$. Clearly, the agreement is quite satisfactory in the region where the flow is laminar. It should be noted, however, that the pressure drop is highly sensitive to minute variation of the diameter of the central vessel. Here, the best agreement was found with a value of $D_2 = 2.1$ cm, which is within the expected uncertainty of the diameter of this vessel.

**Ramification of vessels.** This third experiment is devoted to the study of vascular networks and, in particular, the association of vessels in parallel. In this case, the flow is diverted to the upper line, as shown in the experimental setup (Fig. 1). The screw clamp acting on the silicone tubing that feeds the bifurcation should remain fully open and the shutoff valve inserted in the T junction situated before the silicone tubing must be fully closed.

The students registered the pressure drop that arose between the ends of the association of the three parallel vessels for increasing values of the flow rates. Thanks to the three flowmeters installed in the association, the flow rate along each line could be measured independently. This design offers the possibility of directly observing the effect of the length of vessel on the flow rate, because the three vessel share an almost identical pressure drop between its ends. Moreover, because the flowmeters are equipped with a valve, they offer as well the possibility of investigating the effect that stopping the circulation through a vessel will have on the pressure drop and on flow rates through the other vessels.

Figure 5 shows the results obtained during this experiment. Of course, there is an upper limit on the flow rate that may flow through any of each vessel, which should not exceed ~188 ml/min. This fact will ensure that the regime is always laminar and that Poiseuille’s law and the laws of association of vessels are satisfied. The experimental measurements are compared with the values derived from the theory, where both the pressure drop due to viscous losses (Poiseuille’s law) and the local losses introduced by the presence of the flowmeter have been taken into account, that is,

$$\Delta p = \frac{128 \mu \eta}{\pi D^4} Q + R_{\text{local}} Q = (R_{\text{visc}} + R_{\text{local}}) Q,$$

along each line (where $R_{\text{visc}}$ stands for viscous resistance).
The resistance to the flow of each vessel was then determined from a simple least-square fitting of the experimental data to a straight line passing through the origin, the slope of which corresponds to the resistance. Similarly, the total resistance of the association was determined by taking the total flow rate through the association instead of the individual flows. Figure 6 compares the resistance of each vessel with the ones obtained from Eq. 8 and the total resistance with the value predicted by the law of association of vessels (Eq. 7) based on the theoretical resistance of each vessel separately.

Generation of a critical stenosis. This last experiment is intended to illustrate the consequences of the progressive narrowing of an artery (stenosis) and how it affects the irrigation of a distal organ. This experiment will be also carried out using the upper line of the experimental setup (Fig. 1), but it will be now operated in a different way.

The role of the stenotic artery is played by a highly flexible silicone tubing, where a screw clamp has been previously inserted to control the reduction of its section. The parallel association used in the previous experience, which is connected after the silicone tubing, will simulate the vascular network of the distal organ. To evaluate the effect of stenosis on the flow rate reaching the organ, the effective cross section of the stressed silicone tubing needs to be estimated in advance. For our purposes, the following model for the deformation of the silicone tubing was found to be accurate enough. When the silicone tubing is stressed between the parallel jaws of the clamp, we assume that 1) the wall thickness of the silicone tubing remains unchanged; 2) the inner perimeter of the tubing is constant and equal to its initial circular value; and 3) the stressed cross section adopts the form of a central rectangle bounded by two semicircles, one at each side of rectangle. Under these assumptions, some simple calculations show that the relative reduction of the cross section of the tubing is equal to the square of the relative reduction of the initial diameter, that is

$$\frac{\Delta A}{A_0} = \left(\frac{\Delta D}{D_0}\right)^2,$$

where $\Delta A = A_0 - A$ and $\Delta D = D_0 - D$, with $D_0 = 5$ mm and $A_0 = \pi D_0^2/4$. Distance $D$ is taken as the diameter of the semicircles of the stressed section, which is equal to the height of the central rectangle. In our experiment, the movable jaw of the clamp advances 14.5 mm in 19 turns; therefore, $\Delta D/D_0 = 0.1526n$, where $n$ is the number of turns.

Before the start of the experiment, the shutoff valve installed in the T junction situated before the silicone tubing must be fully opened. This operation opens a low-resistance path that is
associated in parallel with the line of the silicone tubing, thus ensuring that the pressure at the entry of the “stenotic artery” will remain constant. Therefore, the flow rate through the “vascular network” will only be affected by the degree of obstruction of the artery. The students started measuring the total flow through the vascular network when there was no reduction of the cross section of the artery. Then, the obstruction was simulated by turning the screw of the clamp, and its consequences on the total flow rate were registered. The results of this experiment are shown in Figure 7. Clearly, the obstruction of the artery only significantly affects blood flow rate to the organ when the effective reduction of the cross section is already very severe. Here, the degree of reduction must be higher than 75% to reach a critical stenosis. At that moment, any further reduction of the cross section of the vessel will have dramatic consequences on the flow rate to the distal organ. This is so because the resistance to the flow of the vascular network is high and, therefore, the flow will not be altered until the resistance of the obstructed artery becomes comparable to that of the vascular network (3, 6, 7).

CONCLUSIONS

The proposed model gives students the possibility of experimenting and quantitatively validating the fundamental laws of hemodynamics, such as Poiseuille’s law, Bernoulli’s equation, or the association of vessels. These laws are behind many physiological phenomena of the circulatory system, either normal or pathological, such as the local regulation of blood flow, the narrowing of a vessel, the generation of a critical stenosis, etc. Even though precise measurements of the relationship between pressure and flow rate can be done with the proposed experimental setup, its construction remains quite simple and the cost is reasonable. This model has been used for the last 2 years in the physiology courses taught at the Faculty of Biology of the University of Seville, and the students response have been very positive. The degree of satisfaction of students with the experimental setup was evaluated by means of a set of anonymous polls, where the opinion of students about the experimental design and its didactic utility was inquired. According to the results of these polls, the students were enthusiastic with the design of the experience, and they recognize that the proposed experiments were highly useful for the learning of hemodynamic laws.

REFERENCES