Formula sheet:

Taylor series approximation: If a quantity $q = q(x, y, z)$ is measured using some input variables x, y and z which are measured with uncertainties Δx, Δy and Δz, respectively, then Δq can also be find out using the Taylor series approximation given as,

$$\Delta q = \sqrt{\left(\frac{\partial q}{\partial x}\Delta x\right)^2 + \left(\frac{\partial q}{\partial y}\Delta y\right)^2 + \left(\frac{\partial q}{\partial z}\Delta z\right)^2}.$$

Standard deviation: $s = \sqrt{\frac{\sum_i d_i^2}{N}}$.

Standard uncertainty: $\sigma = \sqrt{\frac{N}{N-1}}\,(s)$.

Standard uncertainty in the mean: $\sigma_m = \frac{\sigma}{\sqrt{N}}$.

Weighted average: $x_{avg} = \frac{\sum w_i x_i}{\sum w_i}$

Slope (m) and intercept (c) with equal weights:

$$m = \frac{\sum_i^N y_i(x_i - \bar{x})}{\sum_i^N (x_i - \bar{x})^2} \quad \text{or} \quad m = \frac{N\sum_i^N x_i y_i - \sum_i^N x_i \sum_i^N y_i}{N\sum_i^N x_i^2 - (\sum_i^N x_i)^2} \tag{1}$$

$$c = \bar{y} - m\bar{x} \quad \text{or} \quad c = \frac{\sum_i^N x_i^2 \sum_i^N y_i - \sum_i^N x_i \sum_i^N x_i y_i}{N\sum_i^N x_i^2 - (\sum_i^N x_i)^2}. \tag{2}$$

Uncertainty in slope m and intercept c is given as,

$$u_m = \sqrt{\frac{\sum_i^N d_i^2}{D(N-2)}}, \tag{3}$$

$$u_c = \sqrt{\left(\frac{1}{N} + \frac{\bar{x}^2}{D}\right)\left(\frac{\sum_i^N d_i^2}{(N-2)}\right)}, \tag{4}$$

where,

$$d_i = y_i - mx_i - c,$$
$$D = \sum_i^N (x_i - \bar{x})^2.$$

Slope m and intercept c with unequal weights

The weights are reciprocal squares of the total uncertainty (u_{Total}),

$$w = \frac{1}{u_{\text{Total}}^2}. \tag{5}$$

The mathematical relationships for slope (m) and intercept (c) are,

$$m = \frac{\Sigma_i w_i \, \Sigma_i w_i (x_i y_i) - \Sigma_i (w_i x_i) \, \Sigma_i (w_i y_i)}{\Sigma_i w_i \, \Sigma_i (w x_i^2) - (\Sigma_i w_i x_i)^2}, \tag{6}$$

$$c = \frac{\Sigma_i (w_i x_i^2) \, \Sigma_i (w_i y_i) - \Sigma_i (w_i x_i) \, \Sigma_i (w_i x_i y_i)}{\Sigma_i w_i \, \Sigma_i (w_i x_i^2) - (\Sigma_i w_i x_i)^2}, \tag{7}$$

where x is the independent variable, y is the dependent variable and w is the weight.

The expressions for the uncertainties in m and c are,

$$u_m = \sqrt{\frac{\Sigma_i w_i}{\Sigma_i w_i \, \Sigma_i (w_i x_i^2) - (\Sigma_i w_i x_i)^2}}, \tag{8}$$

$$u_c = \sqrt{\frac{\Sigma_i (w_i x_i^2)}{\Sigma_i w_i \, \Sigma_i (w_i x_i^2) - (\Sigma_i w_i x_i)^2}}. \tag{9}$$