Measurement and uncertainties

Dr. Muhammad Sabieh Anwar
And all my current and former lab colleagues
Developing Physics Laboratory
In the Developing World

http://physlab.lums.edu.pk
CEPE:
Centre for Experimental Physics Education
Indigenization
Goals of CEPE

• Preparing an institutional and national platform for student-driven experimental research in physics.
• Indigenous development of experiments for the laboratory and demonstrations for the classroom.
• Training SSE students in world class experimental physics through required and optional lab courses.
• Sharing expertise, resources and training in physics education with sister institutions and organizations in the country and the region.
Measurement Matters
Accuracy and precision

It is better to be roughly right than precisely wrong!

- John Maynard Keynes
Values of fundamental constants

A measurement is meaningless without uncertainty!

\[ \hbar = 6.62606957 \times 10^{-34} \text{ Js} \]

\[ k_B = 1.3806488(13) \times 10^{-23} \text{ J K}^{-1} \]

\[ e = 1.602176565(35) \times 10^{-19} \text{ C} \]

\[ \sigma = 5.670373 (21) \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \]

\[ c = 2.99792456 ? \text{ ms}^{-1} \]
Measure the GUM Way

Errors are mistakes while uncertainties are not.
Errors are idealized and can never be known.

**Uncertainties are:**
- quantifiable
- There exist formal methods for their determination.
- They are of two kinds: A and B.
- They are transferrable.
Random Errors versus Systematic Errors

Type A Uncertainties

Type B Uncertainties

• Avoid the term “human error” – a human error is a mistake!

Type A
• Evaluated Statistically

Type B
• Evaluated by other means
From a reading(s) emerges data.

From a measurement, one infers physical information about the measurand.
### Rounding off

<table>
<thead>
<tr>
<th>Observed value</th>
<th>Rounded value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.05</td>
<td>3.0</td>
</tr>
<tr>
<td>3.15</td>
<td>3.2</td>
</tr>
<tr>
<td>3.25</td>
<td>3.2</td>
</tr>
<tr>
<td>3.35</td>
<td>3.4</td>
</tr>
<tr>
<td>3.33</td>
<td>3.3</td>
</tr>
<tr>
<td>3.36</td>
<td>3.4</td>
</tr>
</tbody>
</table>

**Concept of precision in numerical values**

- How is 2 different from 2.0?
- How is 2 different from 2.00?
Rounding off a reading

2 V scale
-1.99999 V
+1.99999 V

20 V scale
-19.9999 V
+19.9999 V

Measurement #1
1.67543 V
1.67543 V
1.68 V

Measurement #2
1.66329 V
1.66329 V
1.66 V
Simplest rules for determining precision in a single reading

1. Uniform probability distribution

2. Length of the interval = 1.685 - 1.675 = 0.010

3. Half of the length = 0.010 / 2 = 0.005 V

4. Precision is ± 0.005 V

5. Proportional precision = 0.005 / 1.68 = 0.3%
How does arithmetic affect precision?

\[ R = \frac{8.92}{3.5 \times 10^{-3}} \]
\[ = 2548.571429 \Omega \]

% precision in voltage = 0.06%
% precision in current = 0.01%

Reasonable answer= ?
Example question

Mass of the can = 0.562 kg
Mass of the can with liquid = 1.5778 kg
Mass of the liquid = ?
Another example question

\[ V = \frac{1}{6} \pi D^3 \]

*Precision in the ‘pth’ power of a measurand ‘n’ is |np|.*
Type B Evaluations
Probabilistic interpretation

(1.340 ± 0.005) V
Slightly sophisticated probabilistic interpretation

- \( u_{scale} = \frac{\Delta}{\sqrt{3}} = \frac{0.005}{\sqrt{3}} = 0.0029 \)
- \( u_{instrument} @ 1\% = 0.01 \times 1.34 = 0.0134 \)
- \( u_{combined} = \left( u_{scale}^2 + u_{instrument}^2 \right)^{\frac{1}{2}} = 0.0137 \)

\( (1.34 \pm 0.01) \) V
Erroneous recordings

- $(1.34 \pm 0.005) \text{ V}$
- $(1.34 \pm 2.0) \text{ V}$
- $(1.34 \pm 0.1) \text{ V}$
- $(1.34 \pm 0.12) \text{ V}$
- $(1.34 \pm 0.02) \text{ V}$
- $(1.340 \pm 0.12) \text{ V}$
Scale uncertainty (Finite resolution)

Digital measuring instrument
- \( u_s = \frac{\Delta}{\sqrt{3}} \)

Digitization
- \( u_s = \frac{\Delta}{\sqrt{3}} \)

Analog instruments
- \( u_s = \frac{\Delta}{\sqrt{6}} \)
Infinite precision is not possible!

Digital scale
- 83.6 g
- 83.62 g
- 83.627 g

Analog scale
Reading 83.45 g on an analog scale

\[ \Delta = 0.05 \text{ g} \]
\[ u = 0.02 \text{ g} \]

Estimate of the measurand is 
\[ (83.45 \pm 0.02) \text{ g} \]
assuming a triangular PDF and 68% coverage probability
Coverage probability

Notional equivalence

Mass / g

83.555  83.560  83.565

83.56 - u  83.560  83.56 + u
The Gaussian Distribution:
Mean and Standard deviation

\[ g(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]
Let’s try this out...

(9.38 ± 0.05) V

(9.3 ± 0.5) V
# Absolute accuracy ratings of instruments

## Modern Digital Multimeters

### Specifications

<table>
<thead>
<tr>
<th>Basic Functions</th>
<th>Range</th>
<th>Best Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC Voltage</td>
<td>200mV/2V/20V/200V/1000V</td>
<td>±(0.5%+1)</td>
</tr>
<tr>
<td>AC Voltage</td>
<td>2V/20V/200V/750V</td>
<td>±(0.8%+3)</td>
</tr>
<tr>
<td>DC Current</td>
<td>2mA/20mA/200mA/20A</td>
<td>±(0.8%+1)</td>
</tr>
<tr>
<td>AC Current</td>
<td>20mA/200mA/20A</td>
<td>±(1%+3)</td>
</tr>
<tr>
<td>Resistance</td>
<td>200Ω/2.1kΩ/2kΩ/20kΩ/200kΩ/2MΩ/20MΩ/200MΩ</td>
<td>±(0.8%+1)</td>
</tr>
<tr>
<td>Capacitance</td>
<td>20nF/200nF/2μF/10μF</td>
<td>±(4%+3)</td>
</tr>
<tr>
<td>Temperature (°C)</td>
<td>-40°C ~ 100°C</td>
<td>±(1%+3)</td>
</tr>
<tr>
<td>Temperature (°F)</td>
<td>-40°F ~ 1832°F</td>
<td>±(1%+4)</td>
</tr>
</tbody>
</table>

### Special Functions

- Diode
- Continuity Buzzer
- Data Hold
- Display Backlight: Auto Sensor
- Full Icon Display
- Sleep Mode
- Low Battery Display
- Input Impedance for DC Voltage Measurement: Around 10MΩ
- Max. Display: 1999

### General Characteristics

- **Power**: 9V Battery (6F22)
- **LCD Size**: 59 x 25mm
- **Product Colour**: Red and Grey
- **Product Net Weight**: 275g
- **Product Size**: 165 x 80 x 38.3mm
- **Standard Accessories**: Test Lead, Battery, English Manual, Point Contact Temperature Probe, Test Clip
- **Standard Individual Packing**: Gift Box
- **Standard Quantity Per Carton**: 40pcs
- **Standard Carton Measurement**: 588 x 418 x 340mm (Around 0.085 CBM Per Standard Carton)
- **Standard Carton Gross Weight**: 21kg

Specifications and other information are subject to change without further notice.
Mini Analog MultiMeter

Convenient pocket MultiMeter
With easy to read color coded analog display

Features:
- Easy to read analog display
- Measure AC/DC Voltage, DC Current, Resistance and Decibel
- 5% full scale accuracy
- Battery test on 9V and 1.5V batteries
- Complete with protective holster, test leads and 1.5V AA battery

Easy to read color coded analog display
Type A Evaluations
Repeatability versus Reproducibility
Repeated experiment of sliding a ball down a plane
Statistics of multiple readings

**Mean or Average:** For n measurements of a quantity, the mean is given by,

\[ x = \frac{\sum_{i=1}^{n} x_i}{n} \]

The statistical fluctuations can be quantified by calculating the **standard deviation**, which is given by,

\[ s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}} \]
Take 10 balls out at random. Find their mean and put them back.

Write mean here
How does one calculate $\sigma_m$ and $\sigma$?

$$\sigma_m^2 = \left(\frac{1}{n}\right) \sigma^2$$

$$\sigma^2 = \left(\frac{n}{n-1}\right) s^2$$

$$\sigma_m^2 = \left(\frac{1}{n-1}\right) s^2$$
Worked example of dispersion related uncertainty

<table>
<thead>
<tr>
<th>Resistance R/Ω</th>
<th>Deviation d /mΩ</th>
<th>d² / (mΩ)²</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.615</td>
<td>-10</td>
<td>100</td>
</tr>
<tr>
<td>4.638</td>
<td>13</td>
<td>169</td>
</tr>
<tr>
<td>4.597</td>
<td>-28</td>
<td>784</td>
</tr>
<tr>
<td>4.634</td>
<td>9</td>
<td>81</td>
</tr>
<tr>
<td>4.613</td>
<td>-12</td>
<td>144</td>
</tr>
<tr>
<td>4.623</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>4.659</td>
<td>34</td>
<td>1156</td>
</tr>
<tr>
<td>4.623</td>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td><strong>Mean=4.625</strong></td>
<td></td>
<td><strong>Σ=2442</strong></td>
</tr>
</tbody>
</table>

\[(4.625 \pm 0.007 )\,\Omega\]
Probabilistic interpretation of 
\[(4.625 \pm 0.007) \, \Omega\]

68% confident

Measurement vs. data

From data, we can construct knowledge of the measurand

A mean is chosen. There is a 68% probability that the mean will lie in this region

\[4.625 - 0.007 \quad \text{and} \quad 4.625 + 0.007\]
Significant Differences

Measuring the young’s modulus ‘E’ of steel.

**Newton’s ring experiment**

\( x \times 10^{11} \text{ N/m}^2 \)

- 1.90
- 2.28
- 1.74
- 2.27
- 1.67
- 2.01
- 1.60
- 2.18
- 2.18
- 2.00

**Dial indicator experiment**

\( x \times 10^{11} \text{ N/m}^2 \)

- 2.01
- 2.05
- 2.03
- 2.07
- 2.04
- 2.02
- 2.09
- 2.09
- 2.04
- 2.03
Overview of Type A and B uncertainties

Repeate Measurements
- with changing environment or time for measurement of coefficients
- statistical analysis
  - estimates random errors

Broad Scientific Knowledge
- updated specific information such as look-up data, calibration reports
- intentional change to set-up
- 'accidental' change to set-up, or passage of time
  - reveals systematic error (bias)
    - when corrected for leaves:
      - Type A or Type B uncertainty

combined uncertainty

is Type B for subsequent use
Propagation of uncertainties

What is the length of the rod?
Combining uncertainties

Directly measured quantities

\[ u^2 = u_1^2 + u_2^2 + u_3^2 + \ldots + u_n^2 = \sum_{i} u_i^2 \]

Inferred quantities

\[ u^2(y) = \left( \frac{\partial y}{\partial x_1} \right)^2 u^2(x_1) + \left( \frac{\partial y}{\partial x_2} \right)^2 u^2(x_2) + \left( \frac{\partial y}{\partial x_3} \right)^2 u^2(x_3) + \ldots + \left( \frac{\partial y}{\partial x_n} \right)^2 u^2(x_n). \]
<table>
<thead>
<tr>
<th>Relationship</th>
<th>Error propagation formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z = A + B$</td>
<td>$\Delta Z = \sqrt{(\Delta A)^2 + (\Delta B)^2}$</td>
</tr>
<tr>
<td>$Z = A - B$</td>
<td>$\Delta Z = \sqrt{(\Delta A)^2 + (\Delta B)^2}$</td>
</tr>
<tr>
<td>$Z = AB$</td>
<td>$\frac{\Delta Z}{Z} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$</td>
</tr>
<tr>
<td>$Z = \frac{A}{B}$</td>
<td>$\frac{\Delta Z}{Z} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$</td>
</tr>
<tr>
<td>$Z = A^n$</td>
<td>$\frac{\Delta Z}{Z} =</td>
</tr>
<tr>
<td>Formula</td>
<td>Relative uncertainty</td>
</tr>
<tr>
<td>-------------</td>
<td>-----------------------------------------</td>
</tr>
<tr>
<td>A=L×W</td>
<td>( \frac{u(A)}{A} = \sqrt{\left(\frac{u(L)}{L}\right)^2 + \left(\frac{u(W)}{W}\right)^2} )</td>
</tr>
<tr>
<td>P= ( V^2 / R )</td>
<td>( \frac{u(P)}{P} = \sqrt{\left(\frac{2u(V)}{V}\right)^2 + \left(\frac{u(R)}{R}\right)^2} )</td>
</tr>
</tbody>
</table>
Example question

The velocity $v$, frequency $f$ and wavelength $\lambda$ are related by $v = f \lambda$.

An ultrasonic wave has $f = (40.5 \pm 0.15)\text{Hz}$ and $\lambda = (0.822 \pm 0.022)\text{cm}$. 

Calculate the velocity of the wave?
Paper falling...
Time periods with a stop watch and a microcontroller controlled photodetector...