Q.No.1

The standard error in each dimension is 0.01%.

(a) The volume of a rectangular block is,

\[ V = l_1 l_2 l_3. \]

The error affects the three sides independently. Hence, the standard error in \( V \) can be calculated as,

\[ (\Delta V)^2 = \left( \frac{\partial V}{\partial l_1} \right)^2 (\Delta l_1)^2 + \left( \frac{\partial V}{\partial l_2} \right)^2 (\Delta l_2)^2 + \left( \frac{\partial V}{\partial l_3} \right)^2 (\Delta l_3)^2, \]

\[ = (l_2 l_3)^2 (\Delta l_1)^2 + (l_1 l_3)^2 (\Delta l_2)^2 + (l_1 l_2)^2 (\Delta l_3)^2. \]

Doing a step we get the result,

\[ (\Delta V)^2 = \left( \frac{l_1 l_2 l_3}{l_1} \right)^2 (\Delta l_1)^2 + \left( \frac{l_1 l_2 l_3}{l_2} \right)^2 (\Delta l_2)^2 + \left( \frac{l_1 l_2 l_3}{l_3} \right)^2 (\Delta l_3)^2, \]

implying

\[ \left( \frac{\Delta V}{V} \right)^2 = \left( \frac{\Delta l_1}{l_1} \right)^2 + \left( \frac{\Delta l_2}{l_2} \right)^2 + \left( \frac{\Delta l_3}{l_3} \right)^2, \]

\[ \left( \frac{\Delta V}{V} \right) = \sqrt{(0.01)^2 + (0.01)^2 + (0.01)^2} = 0.017\%, \]

\[ \approx 0.02\%. \]

(b) For temperature variations, all sides are affected equally. Therefore, one can use the formula for volume with equal lengths,

\[ V = l^3. \]

The error in volume will be,

\[ \left( \frac{\Delta V}{V} \right)^2 = \left( \frac{3\Delta l}{l} \right)^2. \]
This result shows that the overall uncertainty can increase, if the errors are not independent nor random.

Q.No.2

(a) The standard deviation of the measured data is $\sigma_u = 10$ m/s.

The standard error in the mean can be found out using the following relationship,

$$\sigma_{\text{mean}} = \frac{\sigma}{\sqrt{n}},$$  \hspace{1cm} (1)

implying,

$$n = \left( \frac{\sigma_u}{\sigma_{\text{mean}}} \right)^2,$$

$$= \left( \frac{10}{3} \right)^2,$$

$$= 11.$$

(b) Let’s see how many times we will repeat the experiment to get a final uncertainty of 0.5 m/s.

Using Equation (1), we get,

$$n = \left( \frac{10}{0.5} \right)^2,$$

$$= 400.$$

Hence we conclude that we need to repeat the measurements 11 and 400 times for minimizing error to 3 m/s and 0.5 m/s, respectively.

Q.No.3

The spring constant $k$ measured by timing the oscillations of a mass $m$ fixed to its end is given as,

$$k = \frac{4\pi^2 m}{T^2}.$$

As the measured masses are not different measurements of the same quantity, therefore the process of averaging can’t work. On the other hand, we are not sure about the uncertainties in our measurements, so we need to calculate $k$ first by combining each value of $m$ with its corresponding period $T$ as given in Table (1).
<table>
<thead>
<tr>
<th>Mass m (kg)</th>
<th>Period T (s)</th>
<th>Spring constant k (N/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.513</td>
<td>1.24</td>
<td>13.17</td>
</tr>
<tr>
<td>0.581</td>
<td>1.33</td>
<td>12.97</td>
</tr>
<tr>
<td>0.634</td>
<td>1.36</td>
<td>13.53</td>
</tr>
<tr>
<td>0.691</td>
<td>1.44</td>
<td>13.16</td>
</tr>
<tr>
<td>0.752</td>
<td>1.50</td>
<td>13.19</td>
</tr>
<tr>
<td>0.834</td>
<td>1.59</td>
<td>13.02</td>
</tr>
<tr>
<td>0.901</td>
<td>1.65</td>
<td>13.07</td>
</tr>
<tr>
<td>0.950</td>
<td>1.69</td>
<td>13.13</td>
</tr>
</tbody>
</table>

Table 1: Measurement of spring constant $k$.

The mean of the measured values of spring constant $k$ is,

$$\bar{k} = \frac{\sum_{i} k_i}{n} = \frac{13.17 + 12.97 + 13.53 + 13.16 + 13.19 + 13.02 + 13.07 + 13.13}{8} = 13.16 \text{ N/m}.$$  

Deviation from the mean value is,

$$d_i = k_i - \bar{k},$$

and the deviations are,

<table>
<thead>
<tr>
<th>$d_i$ (N/m)</th>
<th>$(d_i \text{ (N/m)})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>$1 \times 10^{-4}$</td>
</tr>
<tr>
<td>-0.19</td>
<td>0.0361</td>
</tr>
<tr>
<td>0.37</td>
<td>0.1369</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.03</td>
<td>$9 \times 10^{-4}$</td>
</tr>
<tr>
<td>-0.14</td>
<td>0.0196</td>
</tr>
<tr>
<td>-0.09</td>
<td>$8.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>-0.03</td>
<td>$9 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

The standard deviation, $s$, is,

$$s = \sqrt{\frac{\sum d_i^2}{n}} = \sqrt{\frac{0.2026}{8}} = 0.159 \approx 0.16 \text{ N/m}.$$  

The standard error, $\sigma$, can be find out using the following relationship,

$$\sigma = \sqrt{\frac{n}{n-1}} s = 0.17 \text{ N/m}.$$
We can say that he expected standard error is approximately equal to the standard deviation, $\sigma \approx s$.

Now the standard error in the mean is,

$$\sigma_{m} = \frac{\sigma}{\sqrt{n}}$$

$$= 0.06 \text{ N/m}.$$

Hence the final result can be written as,

$$k = (13.16 \pm 0.06) \text{ N/m}.$$