Answer 1).

Root Mean Square Value:

\[ E_{\text{rms}} = \frac{E_m}{\sqrt{2}}. \]

Where \( E_m \) is the amplitude.

The intensity is given as,

\[ I = S_{\text{avg}} = \frac{1}{c\mu_0} [E^2]_{\text{avg}}. \]

Where \( S \) is the poynting vector.

\[ I = \frac{1}{c\mu_0} \frac{E_m^2}{2}. \]

Therefore,

\[ E_m = \sqrt{2Ic\mu_0}. \]

Now \( I \) is time average value of poynting vector.

\[
\begin{align*}
I &= \frac{\text{Power}}{\text{Area}} \\
   &= \frac{P}{\frac{1}{4\pi r^2}} \\
   &= \frac{100}{4(3.14)} \text{ Wm}^2 \\
   &= 7.96 \text{ Wm}^{-2}.
\end{align*}
\]

Therefore,

\[ E_m = \sqrt{2(7.96) \text{ Wm}^{-2} \times 3 \times 10^8 \text{ ms}^{-1}\mu_0}. \]

Where,
\( \mu_0 = 4\pi \times 10^{-7} \text{ TmA}^{-1}. \)

\[
\begin{align*}
E_m &= \sqrt{6.036 \times 10^3 \text{ WTAs}^{-1}} \\
E_m &= 77.45 \text{ NC}^{-1}.
\end{align*}
\]
Hence, root mean square value of electric field is

\[ E_{\text{rms}} = \frac{77.45}{\sqrt{2}} \]
\[ = 54.7 \text{ NC}^{-1}. \]

**Answer 2).**

Rate of energy per unit area in electromagnetic wave is described by \( \overrightarrow{S} \) (a Poynting vector),

\[ \overrightarrow{S} = \frac{1}{\mu_0} \overrightarrow{E} \times \overrightarrow{B}. \]

Magnitude of \( \overrightarrow{S} = S = \frac{1}{\mu_0} EB \) and amplitude ratio \( c = \frac{E}{B} \).

\[ S = \frac{1}{c\mu_0} E^2. \]

Average of \( \overrightarrow{S} = \text{Intensity of wave } = I. \)

\[ S_{\text{avg}} = \frac{1}{c\mu_0} [E^2]_{\text{avg}} \]
\[ = \frac{1}{c\mu_0} [E_m^2 \sin^2(kx - \omega t)]_{\text{avg}} \]
\[ = \frac{1}{c\mu_0} \frac{E_m^2}{2}. \]

The direction of Poynting vector is along the direction of propagation of the wave.

**Answer 3).**

**Brewster Angle:**

\( \theta_B = \tan^{-1} \left( \frac{n_2}{n_1} \right) \) is for external reflection.
\( \theta_B = \tan^{-1} \left( \frac{n_1}{n_2} \right) \) is for internal reflection.

By Snells law,

\[ n_t \sin \theta_t = n_i \sin \theta_i. \]
For total internal reflection, 

\[ n_i \sin \theta_c = n_t \]  

\[ \sin \theta_c = \frac{n_t}{n_i} \]  

(1)

Where,

\( \theta_t = 90^\circ \)
\( \theta_i = \theta_c \).

Given that, \( \theta_c = 45^\circ \).
Therefore, from equation (1)

\[ \sin 45^\circ = \frac{n_t}{n_i} \]
\[ 0.707 = \frac{n_t}{n_i} \]

The Brewster angle for external reflection is given by,

\[ \theta_B = \tan^{-1} \frac{n_i}{n_t} \]
\[ \theta_B = \tan^{-1} \left( \frac{1}{0.707 \times n_t} \right) \]
\[ \theta_B = 54.73^\circ \].

**Answer 4).**

**Reflectivity:**

From Fresnel equations,

\[ r_\perp = \left( \frac{E_{\text{out}}}{E_{\text{in}}} \right)_\perp = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} \]  

(2)

\[ r_\parallel = \left( \frac{E_{\text{out}}}{E_{\text{in}}} \right)_\parallel = \frac{n_i \cos \theta_t - n_t \cos \theta_i}{n_i \cos \theta_t + n_t \cos \theta_i} \]  

(3)

Where (2) and (3) are amplitude reflection coefficients for incident waves perpendicular and parallel to plane of incidence.

Now,

\[ R = \left( \frac{E_{\text{out}}}{E_{\text{in}}} \right)^2 = r^2. \]
Where R is Reflectivity which is fraction of incident energy that is reflected. Similarly,

\[ R_{\perp} = \left( \frac{E_{or}}{E_{oi}} \right)_{\perp}^2 = r_{\perp}^2, \]

\[ R_{\parallel} = \left( \frac{E_{or}}{E_{oi}} \right)_{\parallel}^2 = r_{\parallel}^2. \]

For normal incidence, \( \theta_i = 0 \) and plane become undefined. Therefore,

\[ R = R_{\parallel} = R_{\perp} = \left( \frac{n_t - n_i}{n_t + n_i} \right)^2 \]

\[ = \left( \frac{1.5 - 1}{1 + 1.5} \right)^2 \]

\[ = 0.04. \]

Hence 4 percent of the light is reflected.

**Answer5).**

From the Fresnel equations, for TE polarized light.

\[ r_{\perp} = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} \]

\[ = \frac{\frac{n_i}{n_t} \cos \theta_i - \cos \theta_t}{\frac{n_i}{n_t} \cos \theta_i + \cos \theta_t}. \]

With the help of Snell's law and by using identity \( \cos^2 \theta + \sin^2 \theta = 1 \), it is clear that

\[ \cos \theta_t = \sqrt{1 - \left( \frac{n_i}{n_t} \sin \theta_i \right)^2}. \]

Therefore, Fresnel equation for \( r_{\perp} \) becomes

\[ r_{\perp} = \frac{\frac{n_i}{n_t} \cos \theta_i - \sqrt{1 - \left( \frac{n_i}{n_t} \sin \theta_i \right)^2}}{\frac{n_i}{n_t} \cos \theta_i + \sqrt{1 - \left( \frac{n_i}{n_t} \sin \theta_i \right)^2}} \]

(4)

\[ r_{\perp} = \frac{\frac{n_i}{n_t} \cos \theta_i - i \sqrt{\left( \frac{n_i}{n_t} \sin \theta_i \right)^2 - 1}}{\frac{n_i}{n_t} \cos \theta_i + i \sqrt{\left( \frac{n_i}{n_t} \sin \theta_i \right)^2 - 1}}. \]

So, we can write
$$r_\perp = \frac{z}{z^*}$$
$$r_\perp^* = \frac{z^*}{z}$$
$$r_\perp r_\perp^* = 1.$$ 

Where,

$$z = e^{-i\alpha}$$

$$z = \frac{n_i}{n_t} \cos \theta_i - i \sqrt{1 - \left(\frac{n_i}{n_t} \sin \theta_i\right)^2}.$$

We have to plot $R$ versus $\theta_i$, therefore from (4)

$$R_\perp = (r_\perp)^2$$

$$= \left[\frac{n_i}{n_t} \cos \theta_i - \sqrt{1 - \left(\frac{n_i}{n_t} \sin \theta_i\right)^2}\right]^2.$$

Answer6).

Brewster angle is often called Polarization angle. The polarization that cannot be reflected at $\theta_i = \theta_B$ is the polarization for which electric field of light waves lies in same plane as incident ray and surface normal, called P-polarized light or $E_\parallel$ polarization. Light with perpendicular polarization is said to be S-polarized light. For beam incident on surface at $\theta_i = \theta_B$, the $E_\parallel$ will not reflected ($r_\parallel = 0$), so the reflected beam will be completely polarized as $E_\perp$.

From Fresnel equations,

$$r_\parallel = \frac{n_i \cos \theta_i - n_t \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_i}.$$  \hspace{1cm} (5)
\[ r_{\perp} = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} \]  

(6)

As,

\[ r_{\parallel} = 0. \]

Therefore, from equation (5) it is clear,

\[ n_i \cos \theta_i = n_t \cos \theta_t. \]

By Snell’s law,

\[ n_i = n_i \frac{\sin \theta_i}{\sin \theta_t} \]
\[ \sin \theta_t \cos \theta_i = \sin \theta_i \cos \theta_t \]
\[ \sin 2\theta_t = \sin 2\theta_i \]
\[ \theta_t = \theta_i. \]

Which contradict the Snell’s law.

So,

\[ 2\theta_t = \pi - 2\theta_i \]
\[ \theta_i = \frac{\pi}{2} - \theta_t. \]

Or

\[ \theta_B = \frac{\pi}{2} - \theta_t. \]

Because \( \theta_B \) is also called polarization angle.

Therefore,

\[ \theta_p = \frac{\pi}{2} - \theta_t. \]

\textbf{Answer7).}

From the Fresnel equations,

\[ r_{\perp} = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} \]
\[ = \frac{\frac{n_i}{n_t} \cos \theta_i - \cos \theta_t}{\frac{n_i}{n_t} \cos \theta_i + \cos \theta_t}. \]
With the help of Snell’s Law and by using identity \( \cos^2 \theta + \sin^2 \theta = 1 \), it is clear that

\[
\cos \theta_i = \sqrt{1 - \left(\frac{n_i}{n_t} \sin \theta_i\right)^2}.
\]

Therefore, Fresnel equation for \( r_\perp \) becomes

\[
\begin{align*}
    r_\perp &= \frac{n_t \cos \theta_i - i \sqrt{n_t \sin \theta_i}^2 - 1}{n_t \cos \theta_i + i \sqrt{n_t \sin \theta_i}^2 - 1} \\
    &= \frac{n_t \cos \theta_i - i \left(\frac{n_t}{n_i} \sin \theta_i\right)}{n_t \cos \theta_i + i \left(\frac{n_t}{n_i} \sin \theta_i\right)}.
\end{align*}
\]

So, we can write

\[
r_\perp = \frac{z}{z^*}.
\]

Where,

\[
z = e^{-i\alpha}.
\]

So, we can write

\[
r_\perp = \frac{e^{-i\alpha}}{e^{i\alpha}} = e^{-2i\alpha} = \cos(2\alpha) - i\sin(2\alpha).
\]

Therefore, phase

\[
\begin{align*}
    \phi_\perp &= \tan^{-1}\left[\frac{-\sin(2\alpha)}{\cos(2\alpha)}\right] \\
    &= \tan^{-1}(-\tan(2\alpha)).
\end{align*}
\]

Therefore, phase shift for \( \text{TE} \) polarized light is \( \phi_\perp \).

\[
\begin{align*}
    \phi_\perp &= -2\alpha \\
    &= -2 \tan^{-1}\left[\frac{\sqrt{n_t \sin \theta_i}^2 - 1}{n_t \cos \theta_i}\right].
\end{align*}
\]
Phase for TE polarization (rad)

\[
\begin{array}{c|cccc}
\text{Angle of incidence (rad)} & -3 & -2.5 & -2 & -1.5 \\
\hline
0.729728 & 0.5 & 1 & 1.5 & 2 \\
\end{array}
\]

Similarly, for \( TM \) polarized light

\[
r_\parallel = \frac{n_i \cos \theta_i - n_t \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_i}
\]

\[
= \frac{n_i \cos \theta_i - \cos \theta_i}{n_i \cos \theta_i + \cos \theta_i}
\]

\[
= \frac{i \frac{n_i}{n_t} \sqrt{(\sin \theta_i \frac{n_i}{n_t})^2 - 1 - \cos \theta_i}}{i \frac{n_i}{n_t} \sqrt{(\sin \theta_i \frac{n_i}{n_t})^2 - 1 + \cos \theta_i}}
\]

\[
= \frac{\cos \theta_i - i \frac{n_i}{n_t} \sqrt{(\sin \theta_i \frac{n_i}{n_t})^2 - 1}}{\cos \theta_i + i \frac{n_i}{n_t} \sqrt{(\sin \theta_i \frac{n_i}{n_t})^2 - 1}}
\]

\[
= \frac{z}{z^*}.
\]

Where,

\[
z = e^{-i\beta}
\]

\[
= \cos \theta_i - i \frac{n_i}{n_t} \sqrt{(\sin \theta_i \frac{n_i}{n_t})^2 - 1}.
\]

By the same argument as for \( r_\perp \),

\[
r_\parallel = -e^{2i\beta}.
\]

Therefore, Phase
\[
= \tan^{-1}\left[\frac{\sin(2\beta)}{\cos(2\beta)}\right] \\
= \tan^{-1}(\tan(2\beta)).
\]

Therefore, phase shift for \textit{TM} polarized light is \(\phi_{\parallel}\)

\[
\phi_{\parallel} = \pi - 2\beta \\
= \pi - 2\tan^{-1}\left[\sqrt{\left(\frac{n_i}{n_t}\sin\theta_i\right)^2 - 1}\right].
\]

\textbf{Phase for TM polarization (rad)}

Therefore, phase difference

\[
\Delta\phi = \phi_{\perp} - \phi_{\parallel} \\
= 2\tan^{-1}\left[-\frac{n_t\cos\theta_i}{n_i\sin^2\theta_i}\sqrt{\left(\frac{n_i}{n_t}\sin\theta_i\right)^2 - 1}\right] - \pi.
\]

Here,

\[
n_i = 1.3. \\
n_t = 1.
\]
Relative phase of TE and TM polarization (rad)

'Answer8).

Polarized light is externally incident on air-glass interface \((n_t > n_i)\).

Given that,

\[ \theta_i = \frac{\pi}{4}, \]
\[ n_i = 1, n_t = 1.5. \]

We have to find the percentages of reflected light \((R)\) and transmitted light \((T)\),

\[ R = 0.5R_{\parallel} + 0.5R_{\perp} \quad (7) \]

Firstly we will evaluate \(R_{\parallel}\) and \(R_{\perp}\),

\[
R_{\perp} = 
\frac{\left[ n_t \cos \theta_i - n_t \sqrt{1 - \frac{n_t^2}{n_i^2} \sin^2 \theta_i} \right]^2}{\left[ n_t \cos \theta_i + n_t \sqrt{1 - \frac{n_t^2}{n_i^2} \sin^2 \theta_i} \right]^2}
\]
\[
= \frac{\left[ 1(0.707) - 1.5 \sqrt{1 - 0.44(0.707)^2} \right]^2}{\left[ 1(0.707) + 1.5 \sqrt{1 - 0.44(0.707)^2} \right]^2}
\]
\[
= \frac{\left[ 0.707 - 1.324 \right]^2}{\left[ 0.707 + 1.324 \right]^2}
\]
\[= 0.0924. \]

Now,
\begin{align*}
R_\parallel &= \left( \frac{n_i \sqrt{1 - \frac{n_i^2}{n_t^2} \sin^2 \theta_i} - n_t \cos \theta_i}{n_i \sqrt{1 - \frac{n_i^2}{n_t^2} \sin^2 \theta_i} + n_t \cos \theta_i} \right)^2 \\
&= \left( \frac{\sqrt{1 - 0.44(0.707)}^2 - 1.5(0.707)}{\sqrt{1 - 0.44(0.707)}^2 + 1.5(0.707)} \right)^2 \\
&= \left( \frac{0.8832 - 1.0605}{0.8832 + 1.0605} \right)^2 \\
&= 0.00831.
\end{align*}

Plug $R_\parallel, R_\perp$ in equation (7),

\begin{align*}
R &= 0.5(0.00831) + 0.5(0.0924) \\
&= 0.0503.
\end{align*}

Therefore, 5.03 percent light is reflected.

\begin{align*}
T + R &= 1 \\
T &= 1 - R \\
&= 1 - 0.0503 \\
&= 0.949.
\end{align*}

Hence, 94.9 percent of incident light is transmitted and 5.03 percent reflected.