a) For a perfectly flat surface, we will have pattern of the alternate dark and bright fringes, any deviation from this pattern (i.e bumps or kinks appear on the screen) will be a measure of error in the flatness of surface being inspected. Therefore, in Michelson interferometer having flat mirrors, we will observe the circular fringes on screen. Otherwise there will be bumps in the fringes.

b) Given that, frequency range is \( \Delta f = (769 \times 10^{12} - 384 \times 10^{12}) = 385 \times 10^{12} \) Hz.

We have to find the coherence time \( \tau_o \) and coherence length \( l_c \),

\[
\tau_o = \frac{1}{\Delta f} \]
\[
= \frac{1}{385 \times 10^{12}} \]
\[
= 2.59 \times 10^{-15} \text{ s.} \]

By using relation between coherence time \( \tau_o \) and coherence length \( l_c \),

\[
l_c = \frac{c \tau_o}{(1)} \]
\[
= 3 \times 10^8 (2.59 \times 10^{-15}) \]
\[
= 7.79 \times 10^{-7} \text{ m} \]

c) Given that, \( \tau_o = 10^{-8} \text{ s} \) and \( \lambda_o = 650 \times 10^{-9} \text{ m} \).

By putting the value of \( \tau_o \) in Equation(1), coherence length \( l_c \) is

\[
l_c = (3 \times 10^3)(10^{-8}) \]
\[
= 3 \text{ m.} \]

For the line width \( \Delta \lambda \),

\[
l_c = \frac{\lambda^2}{\Delta \lambda} \]
\[
\Delta \lambda = \frac{\lambda^2}{l_c} \]
\[
= \frac{(650 \times 10^{-9})^2}{3} \]
\[
= 1.408 \times 10^{-13} \text{ m.} \]

d) The relation between bandwidth and time is,

\[ \Delta f = \frac{1}{\tau_o}, \]

where \( \Delta f \) is range of frequencies. 

From Equation(2), it is clear that faster a wave decorrelates (i.e \( \tau_o \) is small), the longer
the range of frequencies $\Delta f$ of the wave contain. Therefore, a wave containing only a single frequency (monochromatic) is perfectly correlated ($\tau_0 = \infty$) at all times. Laser light is monochromatic, directional and coherent beam while ordinary light is a combination of many different wavelengths (colors). Its monochromaticity implies long coherence length $l_c$. As coherence length is proportional to coherence time ($l_c = c\tau_0$), thus laser has longer coherence time than ordinary bulb light.

e) Given that,

$$n(\lambda) = A + \frac{B}{\lambda},$$

where $A$ and $B$ are positive constants.

$$n(\lambda) = A + \frac{B\omega}{2\pi c}.$$ \hspace{1cm} (3)

By differentiating Equation (3) w.r.t $\omega$,

$$\frac{dn}{d\omega} = \frac{B}{2\pi c} > 0$$ \hspace{1cm} (4)

By the conditions for dispersion,

$$\frac{dn}{d\omega} < 0$$ \hspace{0.5cm} dispersion is anomalous.

$$\frac{dn}{d\omega} > 0$$ \hspace{0.5cm} dispersion is normal.

From Equation (4), it is clear that dispersion is normal.

**Answer 2**

Given that, the length of glass chamber is $p = 25 \times 10^{-2}$ m, fringe shift is $\Delta M = 21$, wavelength of red line is $\lambda = 656.2 \times 10^{-9}$ m and refractive index of air is $n_{air} = 1.000276$. We have to find the refractive index of test gas $n_{gas}$ as air in the chamber is replaced by the test gas. By using the relation between $n_{gas}$ and $n_{air}$,

$$n_{gas} = n_{air} + \frac{\Delta M \lambda}{2p}$$

$$= 1.000276 + \frac{21(656.2 \times 10^{-9})}{2(25 \times 10^{-2})}$$

$$= 1.000276 + 1.378 \times 10^{-5}$$

$$= 1.0030.$$  

**Answer 3**

In the Michelson interferometer, the conditions for bright and dark rings are,

$$2d \cos \theta_N = N\lambda_o$$ \hspace{1cm} for dark rings, \hspace{1cm} (5)
\[ 2d \cos \theta_N = (N + 1)\lambda_o \] 
for bright rings, \hspace{1cm} (6)

where \( N \) is integer.

For the central dark ring, \( N \) is the maximum, \( \theta_N = 0 \) and \( \cos \theta_N = 1 \).

\[ 2d = N\lambda_o. \] 
(7)

Let the \( P \)th dark ring, be

\[ 2d \cos \theta_P = (N - P)\lambda_o. \] 
(8)

By subtracting Equations (7) and (8),

\[ 2d - 2d \cos \theta_P = P\lambda_o \]
\[ 2d[1 - \cos \theta_P] = P\lambda_o. \]

If \( \theta_P \) is small, then \( \cos \theta_P = \sqrt{1 - \sin^2 \theta_P} \approx 1 - \frac{\theta_P^2}{2} \).

Therefore,

\[ 2d[1 - 1 + \frac{\theta_P^2}{2}] = P\lambda_o \]
\[ \theta_P^2 = \frac{P\lambda_o}{d} \]
\[ \theta_P = \sqrt{\frac{P\lambda_o}{d}}, \] 
(9)

is the angular radius of \( P \)th ring, \( P \) is the number of the dark ring, \( d \) is the path difference and \( \lambda_o \) is the wavelength of red light.

Similarly, for the bright rings

\[ \theta_P = \sqrt{(P + 1)\lambda_o \over d} \] 
(10)

Given that, \( d = 20 \times 10^{-6} \text{ m} \) and \( \lambda_o = 632.8 \times 10^{-9} \text{ m} \).

**a)** The angular radius of the first dark ring is,

\[ \theta_1 = \sqrt{\frac{1(632.8 \times 10^{-9})}{20 \times 10^{-6}}} \]
\[ = 0.177 \text{ rad.} \]

**b)** The angular radius of the 10th dark ring is,

\[ \theta_{10} = \sqrt{\frac{10(632.8 \times 10^{-9})}{20 \times 10^{-6}}} \]
\[ = 0.562 \text{ rad.} \]
Given that,

\[ f(x) = \begin{cases} +A & 0 < x \leq a. \\ -A & -a \leq x < 0. \end{cases} \]

The Fourier transform is given by,

\[
F(k) = \int_{-\infty}^{\infty} f(x)e^{-ikx}dx \\
= -A \int_{-a}^{0} e^{-ikx}dx + A \int_{0}^{a} e^{-ikx}dx \\
= \frac{A}{ik} \left[ (1 - e^{ika}) - (e^{-ika} - 1) \right] \\
= \frac{iA}{k} [2 \cos ka - 2] \\
= \frac{-2iA}{k} [1 - \cos ka] \\
= i \left[ \frac{-4A}{k} \sin^2 \frac{ka}{2} \right].
\]

As \( f(x) \) is real and odd, therefore \( F(k) \) will be imaginary (real part of \( F(k) \) is zero) and odd.

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**Fig. Fourier transform of \( f(x) \).**
Given that, wavelength of blue light is $\lambda = 487.99 \times 10^{-9}$ m, thickness of film is $f = 1.648 \times 10^{-6}$ m, refractive index of film is $n_f = 1.555$, angle of incidence $\theta_i = 30^\circ$ and refractive index of air $n_o = 1$.

By Snell’s law,

$$n_o \sin \theta_i = n_f \sin \theta_f$$

$$\theta_f = \sin^{-1} \left( \frac{n_i \sin \theta_i}{n_f} \right)$$

$$= \sin^{-1} \left( \frac{0.5}{1.555} \right)$$

$$= 18.75^\circ.$$

a) We have to find the optical path difference $\Delta_{path}$ due to two reflected rays. Path of the first ray is $n_oAB$.

$$n_oAB = n_o(2f \sin \theta_i \tan \theta_i).$$

Path of the second ray is $n_fAC$.

$$n_fAC = n_f \left( \frac{2f}{\cos \theta_f} \right).$$

Therefore, the optical path difference between two rays $\Delta_{path}$ is
\[ \Delta_{\text{path}} = n_f AC - n_o AB \]
\[ = n_f \left( \frac{2f}{\cos \theta_f} \right) - n_o (2f \sin \theta_i \tan \theta_i) \]
\[ = 2f \left[ \frac{n_f}{\cos \theta_f} - n_o \sin \theta_i \tan \theta_i \right] \]
\[ = 2(1.648 \times 10^{-6}) \left[ \frac{1.555}{\cos 18.75} - \sin 30 \tan 30 \right] \]
\[ = 3.296 \times 10^{-6} \left[ \frac{1.555}{0.946} - 0.228 \right] \]
\[ = 4.46 \times 10^{-6} \text{ m.} \]

b) Phase difference between reflected rays is \( \Delta \phi_{\text{phase}} \),
\[ \Delta \phi_{\text{phase}} = k(\Delta_{\text{path}}) + \Delta \phi_{\text{refl}} \]
\[ = \frac{2\pi}{\lambda} (4.46 \times 10^{-6}) + \pi \]
\[ = 3.14(18.279 + 1) \]
\[ = 60.53 \text{ rads.} \]

c) For the smallest film thickness to give constructive interference, \( \Delta \phi_{\text{path}} \) is
\[ \Delta \phi_{\text{path}} = 2\pi = \frac{4\pi n_f}{\lambda_o} f \cos \theta_f. \]

We have to find the thickness of film \( f \),
\[ f = \frac{\lambda_o}{2n_f \cos \theta_f} \]
\[ = 487.99 \times 10^{-9} \]
\[ = 1.65 \times 10^{-7} \text{ m.} \]

Answer 6

Given that, a medium is disturbed by an oscillation
\[ y = (3\text{cm}) \sin \left( \frac{\pi x}{10\text{cm}} \right) \cos (50\pi t). \]  \( (11) \)
a) The harmonic waves can be represented by the equations below,
\[ y_1 = y_o \sin (kx - \omega t), \]
\[ y_2 = y_o \sin (kx + \omega t), \]
where \( y_0 \) is the amplitude of wave, \( \omega \) is the angular frequency and \( k \) is the wave number. So the resultant wave \( y \) equation is given by,

\[
y = y_0 \sin(kx - \omega t) + y_0 \sin(kx + \omega t) \\
= y_0[\sin(kx - \omega t) + \sin(kx + \omega t)] \\
= 2y_0 \cos(\omega t) \sin(kx).
\]  

(12)

From Equations (11) and (12), it is clear that \( \omega = 50\pi \) and \( k = \frac{\pi}{10cm} \). Amplitude of resultant wave is \( y_0 \),

\[
y_0 = \frac{3}{2}, \\
= 1.5 \text{ cm}.
\]

The frequency \( f \), wavelength \( \lambda \) and speed \( v \) are given by,

\[
f = \frac{\omega}{2\pi} \\
= \frac{50\pi}{2\pi} \\
= 25 \text{ Hz}. \\
\lambda = \frac{2\pi}{k} \\
= \frac{2\pi}{\pi}10\text{cm} \\
= 20 \text{ cm}. \\
v = f\lambda \\
= (25)(20) \\
= 500 \text{ cms}^{-1}.
\]

The direction of propagation of the resultant wave is along the \( x \)-axis.

b) As two waves of same frequency, wavelength and amplitude traveling in opposite directions interfere and generate standing waves. Therefore, Equation (11) represents a standing wave. Also, the time and space parts are distinct. The distance between nodes is \( \frac{\lambda}{2} = 10 \text{ cm} \).

c) For \( x = 5 \text{ cm} \) and \( t = 0.22 \text{ s} \), Equation (12) is

\[
y = 2(1.5 \text{ cm}) \cos(50\pi \cdot 0.22) \sin\left(\frac{\pi}{2}\right) \\
= -3 \text{ cm}.
\]
By taking derivative of Equation(12) w.r.t \( t \), velocity \( v \) is 

\[
v = \frac{dy}{dt} = -2y_0 \omega \sin(\omega t) \sin(kx) \\
= -2(1.5)(50\pi)(0) \sin\left(\frac{\pi}{2}\right) \\
= 0. 
\]

Thus, at \( x = 5 \) cm and \( t = 0.22 \) s displacement is \(-3\) cm, velocity is 0 and acceleration is \(-73947 \) cms\(^{-2}\).

**Answer**

Given that, wavelength of He-Ne laser is \( \lambda = 632.8 \times 10^{-9} \) m, frequency separation between two adjacent modes is \( \Delta f = 150 \times 10^6 \) Hz and reflectivity is \( R = 0.999 \).

a) The coefficient of finesse \( F \) is,

\[
F = \frac{4R}{(1 - R)^2} \\
= \frac{4(0.999)}{(1 - 0.9999)^2} \\
= 3.9 \times 10^6. 
\]

Also, finesse \( \mathcal{F} \) is given by

\[
\mathcal{F} = \frac{\pi \sqrt{F}}{2} \\
= 3.1 \times 10^3. 
\]

b) Now we have to find resolving power \( \mathcal{R} \),

\[
\mathcal{R} = \frac{\lambda}{|\Delta \lambda|}, 
\]

where,

\[
|\Delta \lambda| = \frac{\Delta f \lambda^2}{c} \\
= \frac{(150 \times 10^6)(632.8 \times 10^{-9})^2}{3 \times 10^6} \\
= 2.0021 \times 10^{-13} \text{ m.}
\]
Thus, $R$ is

$$R = \frac{632.8 \times 10^{-9}}{2.0021 \times 10^{-13}} = 3.160 \times 10^6.$$  

c) For the plate spacing $d$,

$$d = \frac{\lambda_o N}{2n},$$

where,

$$N = \frac{R}{\mathcal{F}} = \frac{3.160 \times 10^6}{3.1 \times 10^3} = 1.019 \times 10^3.$$  

Thus, $d$

$$d = \frac{632.8 \times 10^{-9}(1.019 \times 10^3)}{2(1)} = 3.22 \times 10^{-4} \text{ m.}$$

d) The free spectral range $FSR$,

$$FSR = \frac{d_{FSR}}{\lambda_o} = \frac{\lambda^2}{2dn} = \frac{(632.8 \times 10^{-9})^2}{2(3.22 \times 10^{-4})(1)} = 6.209 \times 10^{-10} \text{ m.}$$

e) Now we have to find the minimum resolvable wavelength $\Delta \lambda_{\text{min}}$.

$$\Delta \lambda_{\text{min}} = \frac{\lambda_o}{\mathcal{F}} = \frac{2\lambda_o}{\pi \sqrt{F}} = \frac{2(632.8 \times 10^{-9})}{3.14 \sqrt{3.9 \times 10^6}} = 2.016 \times 10^{-10} \text{ m.}$$
Answers

Given that, wavelength is \( \lambda_o = 541 \times 10^{-9} \) m and line width \( \Delta \omega = 1A^o = 1 \times 10^{-10} \) m.

For a light beam splits in to two equal amplitude parts and interfere after traveling along different paths, exhibit the path difference \( d = 1.50 \times 10^{-3} \) m.

The fringe visibility \( V \) is defined as ratio,

\[
V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}},
\]

where,

\[
I_{\text{max}} = I_1 + I_2 + 2\sqrt{I_1I_2}|\gamma_{12}(\tau)|.
\]

\[
I_{\text{min}} = I_1 + I_2 - 2\sqrt{I_1I_2}|\gamma_{12}(\tau)|.
\]

It follows that,

\[
V = \frac{2\sqrt{I_1I_2}}{I_1 + I_2}|\gamma_{12}(\tau)|.
\]

For the case of equal amplitudes in a two beam interference arrangement, \( |E_1| = |E_2| = |E| \).

So, \( I_1 = <|E|^2> = I_2 \) then the visibility can be written as

\[
V = |\gamma_{12}(\tau)|,
\]

where, \( \gamma_{12}(\tau) \) is the partial degree of coherence.

The normalized autocorrelation function \( |\gamma(\tau)| \) for a quasimonochromatic source is given by,

\[
|\gamma(\tau)| = 1 - \frac{\tau}{\tau_o} \quad \text{for} \quad \tau < \tau_o, \quad (13)
\]

where \( \tau \) is time delay and \( \tau_o \) is the coherence time.

By using relation between line width \( \Delta \lambda \) and coherence time \( \tau_o \),

\[
\tau_o = \frac{\lambda^2}{c\Delta \lambda} = \frac{(541 \times 10^{-9})^2}{(3 \times 10^8)(1 \times 10^{-10})} = 9.75 \times 10^{-12} \text{ s}.
\]

Time delay \( \tau \),

\[
\tau = \frac{\lambda}{c} = \frac{541 \times 10^{-9}}{3 \times 10^8} = 1.803 \times 10^{-15} \text{ s}.
\]
By plugging the values of $\tau$ and $\tau_o$ in Equation (13), $V$ is

$$V = 1 - \frac{\tau}{\tau_o}$$

$$= 1 - \frac{1.803 \times 10^{-15}}{9.75 \times 10^{-12}}$$

$$= 1 - 1.849 \times 10^{-4}$$

$$\approx 0.99$$

The coherence length is given by,

$$l_c = c\tau_o$$

$$= (3 \times 10^8)(9.75 \times 10^{-12})$$

$$= 2.925 \times 10^{-3} \text{ m.}$$

The conditions for interference are,

$$d < l_c,$$

$$\tau < \tau_o.$$

As the path difference is doubled i.e $d' = 2d = 3 \times 10^{-3}$ m, then $(3 \times 10^{-3} > 2.92 \times 10^{-3})$ the interference condition $d < l_c$ is not satisfied. Thus, interference will barely occur.