Answer 1. C

Answer 2. C

Answer 3. C

Answer 4.

Dielectrics have bound charges. Therefore, the relative permittivity for dielectrics is given by,

$$\varepsilon_r(\omega) = 1 + \frac{N e_i}{m \varepsilon_0} \left[ \frac{1}{(\omega_0^2 - \omega^2) - i\gamma\omega} \right].$$

For metals (have free charges), relative permittivity is,

$$\varepsilon_r(\omega) = 1 - \frac{N e_i}{m \varepsilon_0} \left[ \frac{1}{\omega^2 + i\gamma\omega} \right],$$

$$\varepsilon_r(\omega) = n^2 = 1 - \frac{N e_i}{m \varepsilon_0} \left[ \frac{1}{\omega^2 + i\gamma\omega} \right],$$

where conductivity,

$$\sigma(\omega) = \frac{\sigma(0)}{1 - i\omega\tau} = \frac{Ne_2\tau/m}{1 - i\omega\tau},$$

and $\tau$ is the relaxation time, $\varepsilon_r(\omega)$ is frequency dependent.

$$\varepsilon_r(\omega) = 1 + i \frac{\sigma(\omega)}{\varepsilon_0\omega} = 1 + i \frac{\sigma(0)}{\varepsilon_0\omega(1 - i\omega\tau)}.$$ \(\varepsilon_r(\omega)\) depends on frequency and has its maximum value at $\omega = 0$. The relationship $n^2 = \varepsilon_r$ holds only for high frequencies, such as optical frequencies. At small frequencies (such as static fields), the polarizability of the water molecules has contributions not only from bound charges, but also from the polarizability of the molecule (water is dipolar) and hence the dielectric strength is much higher than can simply be expected from the bound charges. The quoted dielectric constant of 80 is at static fields whereas the quoted refractive index is for optical frequencies.

Answer 5.
The plasma frequency $\omega_p$ is given by,

$$\omega_p^2 = \frac{Ne^2}{m\varepsilon_0},$$

where $m$ is the mass of the electron, $N$ is the number of charge carriers (electrons) per unit volume. In our case,

$$\omega_p = 2\pi \times 3 \times 10^6 \text{ rads}^{-1}.$$ 

The free electron density is given by,

$$N = \frac{\omega_p^2 m \varepsilon_0}{e^2} = \frac{3.549 \times 10^{14} \times 9.11 \times 10^{-31} \times 8.85 \times 10^{-12}}{2.56 \times 10^{-38}} = 1 \times 10^{11} \text{ m}^{-3}.$$ 

**Answer 6.**

Given that, the wavelength of CO$_2$ laser is $10,600 \times 10^{-9}$ m and power is $3 \times 1000$ W we have to find the intensity of radiation when beam is focused to an area of $10^{-5}$ cm$^2$.

$$I = \frac{\text{Power}}{\text{Area}} = \frac{3 \times 1000}{10^{-5} \times 10^{-4}} = 3 \times 10^{12} \text{ Wm}^{-2}. $$

For the electric field amplitude $E_0$, we use the relation,

$$E_0 = \sqrt{2Ic\mu_0} = 47 \times 10^6 \text{ NC}^{-1},$$

where, $c = 3 \times 10^8 \text{ ms}^{-1}$ and $\mu_0 = 4\pi \times 10^{-7} \text{ TmA}^{-1}$.

**Answer 7.**

Polarization of light is described by specifying the direction of the electric field vector. For P-polarized light, the electric field is parallel to the plane of incidence and perpendicular to the plane of the interface.

$$\vec{E}(x, y, z, t) = E_0 e^{i(k \cdot r - \omega t + \phi)}.$$
Therefore, for P-polarized light,

\[ \overrightarrow{k} = k_x \hat{i} + k_y \hat{j}, \]
\[ \overrightarrow{E}(r,t) = (E_{0x} \hat{i} + E_{0y} \hat{j})e^{i(k_x x + k_y y - \omega t + \phi)} \]
and \[ |\overrightarrow{K}| = \frac{2\pi}{\lambda}. \]

Therefore, the vectorial part of the electric field is \((E_{0x} \hat{i} + E_{0y} \hat{j})\) and the scalar part is \(e^{i(k_x x + k_y y - \omega t + \phi)}\).

**Answer 8.**

Given that for sea water, the refractive index (real part) is,

\[ n_R = 1.33. \]

According to the given condition, sea water absorbs 99.8 percent of red light in a distance \(z\) of 10 m. Therefore,

\[ \frac{I(z)}{I_0} = 1 - 0.998 = 0.002. \]

Wavelength for red light is \(\lambda = 700 \times 10^{-9} \text{ m}\) and we have to find complex refractive index \(n_i\). By using Lambert’s Beer relationship for reduction in intensity,

\[ I = I_0 e^{-\beta z}, \]

where the extinction coefficient \(\beta\) is,

\[ \beta = \frac{2\omega}{c} n_i. \]  \hspace{1cm} (2)

From Eq (1) and Eq (2) we have,

\[ \frac{I}{I_0} = e^{-\beta z} \]
\[ -\beta = \frac{1}{z} \ln(0.002) \]
\[ \beta = 0.621 \text{ m}^{-1}. \]

Using Eq (2),

\[ n_i = \frac{\beta c}{2\omega} = \frac{\beta c \lambda}{4\pi c} = \frac{0.621 \times 700 \times 10^{-9}}{4 \times 3.14} = 3.46 \times 10^{-8}. \]
Hence the complex refractive index is \((n_R + i n_i) = (1.33 + i3.46 \times 10^{-8})\) and the complex dielectric constant (with the given assumption) is,

\[
\varepsilon_r = (n_R + i n_i)^2 \\
= (1.768 - 1.19 \times 10^{-15}) + i9 \times 10^{-8} \\
= 1.76 + i9 \times 10^{-8}.
\]

**Answer 9.**

a) Output pulse is delayed because light does not travel at infinite speed. Rather, it has a finite speed in any medium. Considering a refractive index of approximately 1.4, the speed of propagation of light (phase velocity) in the silica fiber is,

\[
\frac{c}{n} \approx \frac{3 \times 10^8}{1.4} = 2.14 \times 10^8 \text{ ms}^{-1}.
\]

It takes approximately \(4.7 \times 10^{-6}\) s for light to travel down a length of 1 Km.

b) The time spread of the output pulse is larger because of the dispersion of the medium. The refractive index depends on the wavelength or equivalently, the wave number \(k\). The input pulse has a spread of \(k's\), some components lag behind while others lead. As these components travel through a length of 1 Km, the pulses spread out in time.

c) For normal dispersion, \(n \propto \omega\). Because blue has shorter \(\lambda\), so higher \(\omega\) and higher \(n\). Therefore, blue travels faster than red.

d) 
\[
n = 1.45248 \text{ at } \lambda = 850 \times 10^{-9}\text{ m}, \text{ and } n = 1.44427 \text{ at } \lambda = 1500 \times 10^{-9}\text{ m}
\]
We have to find difference in time \(\Delta t\) for gaussian pulses centred at \(850 \times 10^{-9}\text{ m}\) and \(1500 \times 10^{-9}\text{ m}\).

\[
\Delta t = t_2 - t_1 \\
= \frac{x_2}{v_2} - \frac{x_1}{v_1}.
\]

Therefore,

\[
n_1 = 1.45248, \quad \lambda_1 = 850 \times 10^{-9} \text{ m},
\]

\[
v_1 = \frac{c}{n_1} = \frac{3 \times 10^8}{1.45248} = 206.5432 \times 10^6 \text{ ms}^{-1}.
\]
Similarly,

\[ n_2 = 1.44427, \quad \lambda_2 = 1500 \times 10^{-9} \text{ m}. \]

\[ v_2 = \frac{c}{n_2} = \frac{3 \times 10^8}{1.44427} = 207.717 \times 10^6 \text{ ms}^{-1}. \]

Therefore,

\[ t_1 = \frac{x_1}{v_1} = \frac{1000}{206.5432 \times 10^6} \text{ s} = 4.841 \times 10^{-6} \text{ s}. \]

Similarly, for the pulse centred at 1500 \( \times 10^{-9} \text{ m}, \)

\[ t_2 = \frac{x_2}{v_2} = \frac{1000}{207.717 \times 10^6} \text{ s} = 4.8142 \times 10^{-6} \text{ s}. \]

Thus,

\[ \Delta t = t_1 - t_2 = 4.841 \times 10^{-6} - 4.8142 \times 10^{-6} \text{ s} = 2.7402 \times 10^{-8} \text{ s}. \]