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Birefringence of cellotape: Jones representation and experimental analysis

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Abstract

In this paper, we analyse a simple experiment to study the effects of polarized light. A simple optical system composed of a polarizer, a retarder (cellotape) and an analyser is used to study the effect on the polarization state of the light which impinges on the setup. The optical system is characterized by means of a Jones matrix, and a simple procedure based on Jones vectors is used to obtain an expression for the intensity after the light passes through the optical system. The light intensity is measured by a photodetector and the expression obtained theoretically is experimentally validated. By fitting the experimental intensity data, the value of the retardation introduced by the retarder can also be obtained.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Light is a transversal electromagnetic wave and can therefore be polarized in different ways. Students are usually familiarized with linear polarization, the use of linear polarizers and Malus's law. Nevertheless, it is not easy for them to understand concepts such as circular or elliptic polarized light, birefringence or the action of retarders. Birefringence was discovered in 1669 by Erasmus Bartholinus (1625–1698) using calcite, which is called Iceland spar in

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older textbooks [1, 2]. This phenomenon was first explained in 1690 by Huygens, who used the concept of an elliptical secondary wave [1, 2]. As described in Guenther's book [2], Bartholinus believed that the behaviour of calcite was a special case of refraction and thus called the effect 'double refraction'. It is important to remember that 'refringence' is an archaic synonym for 'refraction' and so its derivative, 'birefringence', is the term commonly used for 'double refraction' [2]. Retarders are devices which modify the polarization of an incident wave changing the relative phase of two orthogonally polarized waves that form the incident wave, and birefringence can be used to produce the desired phase change [2]. To understand how birefringent retarders work, we consider a uniaxial, birefringent plate cut with the optic axis parallel to the plane of the plate. When a beam of linearly polarized light passes normally through this plate, the wave can be decomposed into two components: one of them linearly polarized parallel to the optic axis of the plate and the other polarized perpendicular to this axis [2, 3]. These waves are the ordinary and extraordinary waves in the birefringent plate and they propagate parallelly inside the crystal at different velocities. Hence, one wave is retarded relative to the other. If the retarder plate has a thickness d, these two waves emerge from the crystal with a phase difference given by the following expression [2]:

$$\Gamma = \frac{2\pi}{\lambda_0} \Delta n d \tag{1}$$

where λ_0 is the wavelength of the incident light in air and Δn is the difference in the refractive indices of the crystal for these two components [3]. A retarder can be used to rotate the direction of linear polarization or convert linearly polarized light into elliptically polarized light [2, 4].

The operation of a polarizer or retarder can be described using Jones matrices. An easy way of presenting monochromatic polarized light is by means of an electric vector written in column representation, $\mathbf{J}$, developed by R Clark Jones in 1941 and called the Jones vector. In a similar manner, an optical system which transforms a $\mathbf{J}_1$ state into a $\mathbf{J}_2$ state can be written in terms of a 2×2 matrix, known as the Jones matrix, $\mathbf{T}$, and then it is possible to write $\mathbf{J}_2 = \mathbf{T}\mathbf{J}_1$. Working with matrices has the advantage that the action of optical systems can be represented by a matrix given by the product of the Jones matrices of the different elements which make up the system. This method has great potential for students since a large number of optical systems can easily be analysed by performing matrix multiplication.

This is a general method for presenting polarized light and birefringence using an experimental device composed of a birefringent plate placed between a polarizer and an analyser. After passing through the polarizer, the light is linearly polarized in the direction given by the transmission axes of the polarizer. The electric field can be decomposed into two orthogonal axes x and y, and after light passes through the retarder (birefringent plate), a phase shift $\mathbf{\Gamma}$ is introduced between the x component, E_x, and the y component, E_y, of the electric field. The light then passes through the analyser with the transmission axes forming an angle θ with the transmission axes of the polarizer. The intensity as a function of the angle θ can be measured and the value of the retardation angle $\mathbf{\Gamma}$ can be obtained by making a theoretical fit to the experimental data.

In this paper, we analyse, theoretically and experimentally, the birefringence of a retarder plate. Following Blanco *et al* [3], we use strips of cellophane tape as the retarder plate and verify that this material behaves like a uniaxial crystal. The use of cellotape in polarization demonstrations has already been reported in several educational papers. The use of cellotape to study birefringence was proposed by Feymnan [3, 5] and an interesting photograph of a strip of Scotch tape placed between two crossed polarizers can be found in the third edition of Hecht's book on optics [6]. Babic and Cepic [7] use cellophane to generate colours and Velasquez *et al* [8] employ the Jones representation to analyse cellophane tape retarders and the

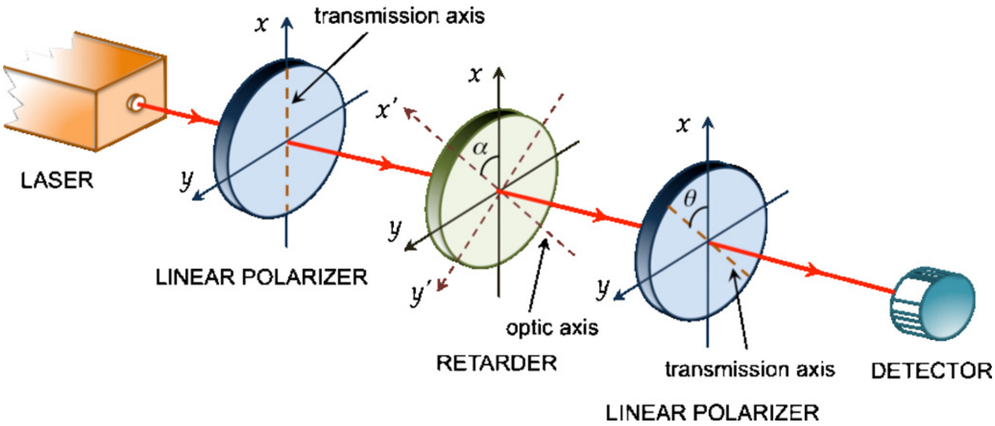

Figure 1. Experimental setup.

experimental measurements are carried out using a broadband light source and a spectrometer. In our experiments, the cellotape is placed between two polarizers and illuminated with monochromatic light at different wavelengths. Measuring the intensity through the system, it is possible to obtain information about the characteristics of the cellotape as a retarder plate. The experiment described, although very simple, has great potential since different concepts relating to polarization of light can be studied easily. In this paper, we propose another simple experiment which is based on the one described, although some changes have been made. This enriches the laboratory experience and permits the students to practice using matrix representation. I think this work can be interesting for experimental demonstration to undergraduate students in courses on optics.

2. Theoretical analysis

Consider an experimental device such as that represented in figure 1. The device is composed of a birefringent crystal plate placed between a polarizer and an analyser. After passing through the polarizer, the light is linearly polarized in the direction given by the transmission axes of the polarizer. The electric field can be decomposed into two orthogonal axes x and y, and after light passes through the retarder (birefringent plate), a phase shift $\boldsymbol{\Gamma}$ is introduced between the x component, E_x, and the y component, E_y, of the electric field. The light then passes through the analyser and is again linearly polarized. In this section, the optical system (polarizer–retarder–analyser) is characterized by obtaining its Jones matrix. As can be seen from the figure, the transmission axis of the polarizer coincides with the x axis, the angle introduced by the retarder will be $\boldsymbol{\Gamma}$, and its optical axis forms an angle α with the x axis. Finally, the transmission axis of the analyser forms an angle θ with the x axis. The Jones matrix of the system can be calculated as follows:

$$\mathbf{H} = \mathbf{T}_3\mathbf{T}_2\mathbf{T}_1 \tag{2}$$

where $\mathbf{T}_3$ is the matrix corresponding to the analyser, $\mathbf{T}_2$ the matrix of the retarder and $\mathbf{T}_1$ that of the linear polarizer. For a linear polarizer with the transmission axis along the x axis, the Jones matrix is given as follows:

$$\mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}. \tag{3}$$

On the other hand, the transmission axis of the analyser forms an angle θ with the x axis, whereas the optical axis of the retarder is also allowed to form an angle α with the x axis. If a Jones matrix, $\mathbf{T'}$, is obtained in a system $x'y'$ which is rotated through an angle β with respect to the axis xy, the matrix $\mathbf{T}$ in system xy is given by

$$\mathbf{T} = \mathbf{R}(\beta)\mathbf{T}'\mathbf{R}(-\beta) \tag{4}$$

where $\mathbf{R}(\beta)$ is the 2×2 rotation matrix given by

$$\mathbf{R}(\beta) = \begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix} \tag{5}$$

Therefore, the Jones matrix $\mathbf{T}_3$ corresponding to the analyser, which is a linear polarizer with a transmission axis making an angle θ with the x axis, is

$$\begin{aligned} \mathbf{T}_3 = \mathbf{R}(\theta)\mathbf{T}'_3\mathbf{R}(-\theta) &= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}. \end{aligned} \tag{6}$$

The matrix $\mathbf{T}'_2$ corresponding to the wave retarder in the system $x'y'$ (because we measure intensities, we can assume the fast axis is along the x' direction) is

$$\mathbf{T}'_2 = \begin{bmatrix} 1 & 0 \\ 0 & \exp(-\mathrm{i}\Gamma) \end{bmatrix} \tag{7}$$

and then, in the xy system, it is possible to write

$$\mathbf{T}_2 = \mathbf{R}(\alpha)\mathbf{T}'_2\mathbf{R}(-\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \exp(-\mathrm{i}\Gamma) \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}. \tag{8}$$

The final matrix of the system (polarizer–retarder–analyser) can be calculated as follows:

$$\begin{aligned} \mathbf{H}(\alpha,\theta) = &\begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \exp(-\mathrm{i}\Gamma) \end{bmatrix} \\ &\times \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned} \tag{9}$$

The matrix $\mathbf{H}$ can be simplified and we obtain

$$\mathbf{H}(\alpha,\theta) = [\cos\alpha\cos(\alpha+\theta) + \exp(-\mathrm{j}\Gamma)\sin\alpha\ \sin(\alpha+\theta)] \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix}. \tag{10}$$

Once the matrix which characterizes the optical system is obtained, the output Jones vector $\mathbf{J}_2$ is calculated by multiplying $\mathbf{H}$ by the Jones vector of the incident light

$$\mathbf{J}_1 = \begin{bmatrix} A_{1x} \\ A_{1y} \end{bmatrix}. \tag{11}$$

The output intensity can be obtained using the equation

$$I(\alpha,\theta) = \tfrac{1}{2}\varepsilon_0 c\ (|A_{2x}|^2 + |A_{2y}|^2) \tag{12}$$

where ε_0 is the permittivity of free space, c is the velocity of light and

$$\mathbf{J}_2 = \begin{bmatrix} A_{2x} \\ A_{2y} \end{bmatrix} \tag{13}$$

where $\mathbf{J}_2 = \mathbf{H}\mathbf{J}_1$. After some manipulations, and taking into account that the polarization of the light incident on the first polarizer is given by $\mathbf{J}_1 = \binom{1}{0}$, the calculated intensity can be written as follows:

$$I(\alpha, \theta) = \frac{1}{2} I_0 \left[1 + (\cos^2 2\alpha + \cos\Gamma \sin^2 2\alpha) \cos 2\theta + \sin^2 \left(\frac{\Gamma}{2} \right) \sin 4\alpha \sin 2\theta \right] \quad (14)$$

where I_0 denotes the maximum intensity transmitted by the whole system and for the experimental measurements, I_0 will also take into account the losses that occur in the different elements of the system (polarizer, cellotape and retarder). The intensity at the output of the optical system is, then, a function of the angle θ between the transmission axis of the analyser and the plane of polarization of the incident light; of α, which is the angle formed by the optical axis of the retarder and the x axis; and of $\mathbf{\Gamma}$, the retardation angle introduced by the retarder. If we define $x = \cos 2\theta$, $y = I$ and $y_0 = I_0$, equation (14) can be written as follows:

$$x^2 + 4\left(y - \frac{y_0}{2} \right)^2 - 4(\cos^2 2\alpha + \cos\Gamma \sin^2 2\alpha) x \left(y - \frac{y_0}{2} \right) = \sin^4 \left(\frac{\Gamma}{2} \right) \sin^2 4\alpha, \quad (15)$$

which, in general, corresponds to an ellipse in the xy plane whose centre is the point with coordinates $(0, I_0/2)$. Then, if I is plotted as a function of $\cos 2\theta$, an ellipse is obtained. An interesting case to study is when $\alpha = 45°$. In this case, from equation (14), it is easy to verify that there is a linear relation between the intensity, I, and $\cos 2\theta$

$$I = \tfrac{1}{2} I_0 (1 + \cos\Gamma \cos 2\theta) \quad (16)$$

and the ellipse in equation (15) is transformed into a straight line

$$y = mx + n \quad (17)$$

where

$$m = \tfrac{1}{2} I_0 \cos\Gamma \quad (18)$$

and

$$n = \tfrac{1}{2} I_0. \quad (19)$$

From equations (18) and (19), it is possible to obtain the value of $\cos\Gamma$ from intensity measurements as follows:

$$\cos\Gamma = \frac{m}{n}. \quad (20)$$

This allows us to obtain the value of the phase $\mathbf{\Gamma}$ knowing the values of m and n, which can be calculated from a least-squares fit to the experimental data of the intensity as a function of $\cos 2\theta$, when $\alpha = 45°$.

3. Experimental results

As mentioned above, different contributions of great interest may be found in the literature describing simple experiments which enable the students to understand how a retarder works and determine the retardation introduced by a uniaxial crystal [3]. As in these references, we used cellotape as the birefringement material and an optic system consisting of a linear polarizer and an analyser, both mounted on rotation graduated stages, with the cellotape placed between them and also mounted on a graduated rotation stage. The different elements making up the system are located along the z axis which coincides with the direction of propagation of light. After light passes through the system, we measured the irradiance by means of a detector. Figure 2 shows the graduated rotation stage where the cellotape was placed. We used

Figure 2. Photograph of the cellotape on the rotation stage.

Table 1. Values of m, n and $\cos\Gamma$, which can be calculated from a least-squares fit to the experimental data of the intensity as a function of $\cos 2\theta$ for $\alpha = 45°$.

λ_0 (nm)	m	n	$\cos\Gamma$
442.0	-9.3 ± 0.4	135.1 ± 1.3	-0.0686 ± 0.0024
543.5	-87 ± 3	106 ± 3	-0.825 ± 0.010
594.0	-88 ± 3	92 ± 3	-0.958 ± 0.009
632.8	-15.5 ± 0.4	15.6 ± 0.4	-0.995 ± 0.005

four continuous-wave (cw) lasers of wavelengths 632.8 nm (red HeNe laser), 594 nm (yellow HeNe laser), 543.5 nm (green HeNe laser) and 442 nm (blue HeCd laser). First, we used equation (16) to adjust the value of $\alpha = 45°$. To do this, we selected $\alpha = 45°$ on the rotation stage where the strip of the cellotape is placed. Then we measured the intensity as a function of $\cos\theta$ and slightly modified the value of α with the help of the micrometric screw to obtain a linear relation between I and $\cos 2\theta$, as shown in equation (17). This reference is considered the direction of the optic axis of the cellotape for $\alpha = 45°$ and it allowed us to easily select the other values of the angle α. We can also find the neutral axes of the retarder in a much simpler way by placing it between crossed polarizers and seeking the null transmission.

In our first experiments, for each wavelength, we chose a value of α of 45° and measured the intensity I through the whole system rotating the angle θ of the analyser between 0° and 180°, and taking measurement every 5° (37 measurements). Then we plotted the values of I as a function of $\cos 2\theta$, with $\alpha = 45°$, and obtained the values of m and n, which can be calculated from a least-squares fit to the experimental data of the intensity as a function of $\cos 2\theta$. Table 1 and figure 3 show the results obtained. From this figure, it is clear that for $\alpha = 45°$, there is a linear relation between I and $\cos 2\theta$.

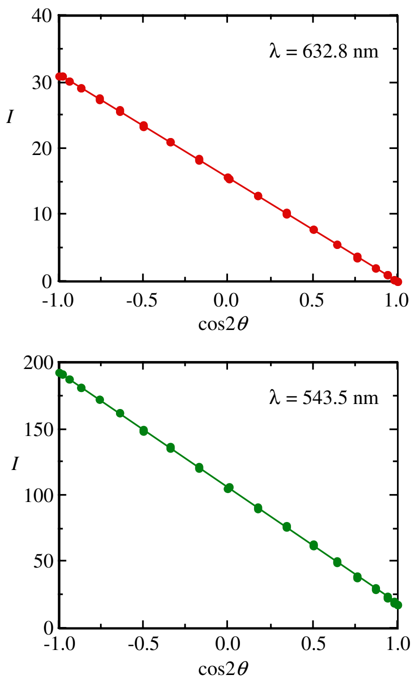

Figure 3. Measured values of the intensity (dots) as a function of $\cos 2\theta$ for $\alpha = 45°$ and linear fit (continuous lines) for the four wavelengths considered.

However, the cosine alone is not sufficient to define the angle, and so the value of the phase difference Γ cannot be obtained from this value alone [3]. To obtain the true value of the phase difference, we proceeded as follows. If Γ is the true value of the phase difference, its value will be that given in equation (1), where we can only ensure that $\Delta n = |n_e - n_o|$, with n_e and n_o being the refractive indices for the extraordinary and ordinary waves, respectively. Even though Δn itself is wavelength dependent, we disregarded any variation of Δn with wavelength because it is very small, and then it may be assumed that the retardation angle Γ is inversely proportional to the wavelength λ_0. We verified that this supposition is approximately correct for the interval of wavelengths considered. Knowing the value of $\cos\Gamma$ from the least-squares fit, we obtain the smallest possible value Γ_1 of the phase difference Γ, which is given by

$$\Gamma_1 = \cos^{-1}(\cos\Gamma) \qquad \text{with} \quad 0 \leqslant \Gamma_1 < \pi. \tag{21}$$

Then, the relation between the true and the smallest possible values for the phase difference can be written as follows:

$$\Gamma = 2\pi N \pm \Gamma_1 \qquad \text{with} \quad 0 \leqslant \Gamma_1 < \pi \tag{22}$$

where N is a non-negative integer. In order to resolve the ambiguities in the values of the phase shifts, we consider equation (20), which allows us to write (disregarding any variation of Δn with wavelength)

$$\Gamma_{\text{blue}} > \Gamma_{\text{green}} > \Gamma_{\text{yellow}} > \Gamma_{\text{red}} \tag{23}$$

and

$$\frac{\Gamma_j}{\Gamma_{\text{blue}}} = \frac{\lambda_{\text{blue}}}{\lambda_j}. \tag{24}$$

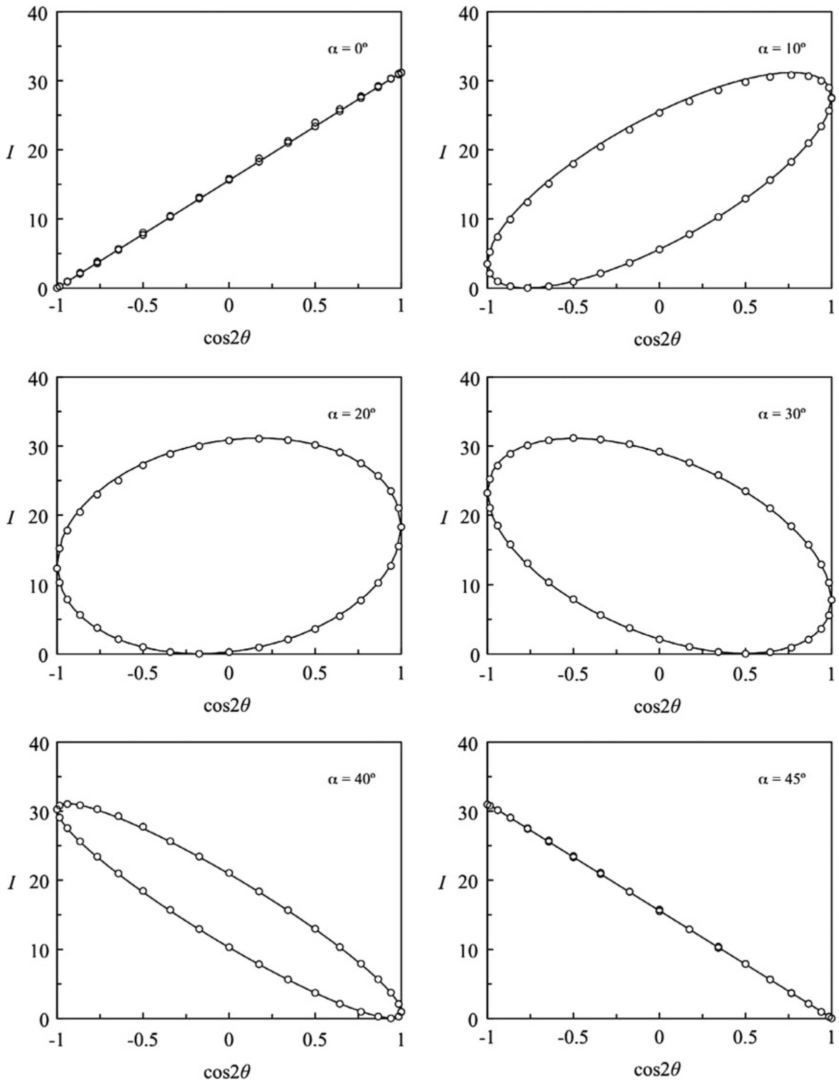

Figure 4. Measured intensity as a function of $\cos 2\theta$ for a wavelength of 632.8 nm: experimental measurements (dots) and theoretical values (continuous line) obtained considering $I_0 = 15.6 \pm 0.4$ (au) and $\cos\Gamma = -0.995 \pm 0.005$ in equation (13).

From equations (20)–(24), it may be easily verified that a value of $N = 1$ and the minus sign must be selected in equation (22). In table 2 we present the different steps followed to obtain the true value of the retardation angle.

In a subsequent experiment, and for a wavelength of 633 nm, we measured the intensity as a function of the angle θ but for different values of α (0°, 10°, 20°, 30°, 40° and 45°). In figure 4 we plotted the measured intensity as a function of $\cos 2\theta$. In this figure, the dots represent the experimental values, whereas the continuous lines correspond to the theoretical values obtained using equation (13) with the values of I_0 and $\cos\Gamma$ obtained previously. We can see in this figure the different types of ellipses obtained. Figure 5 shows the measured intensity as

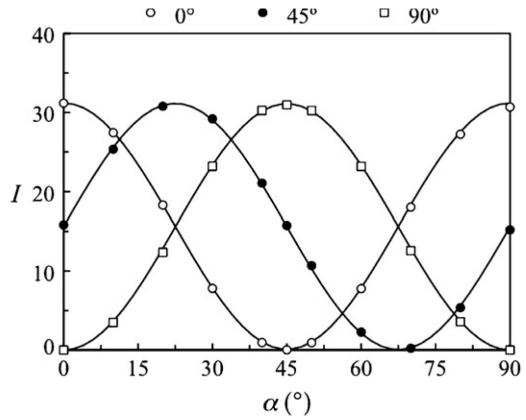

Figure 5. Measured intensity as a function of α for different values of θ and a wavelength of 632.8 nm.

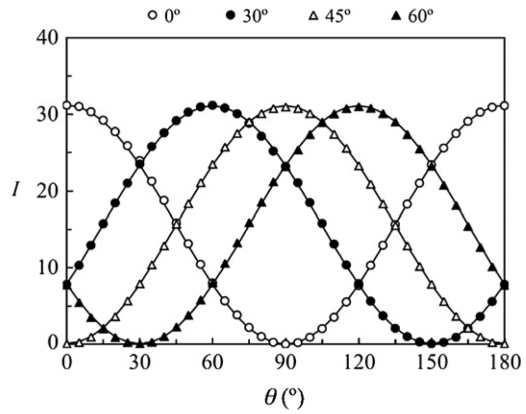

Figure 6. Measured intensity as a function of θ for different values of α and a wavelength of 632.8 nm.

Table 2. Determination of the true value of the retardation angle.

λ_0 (nm)	Γ_1 (deg)	$(\Gamma_j/\Gamma_{blue})_{theoretical} = \lambda_j/\lambda_{blue}$	$(\Gamma_j)_{experimental}$ (deg)	$(\Gamma_j/\Gamma_{blue})_{experimental}$
442.0	93.9 ± 0.2	1	266.1 ± 0.2	1
543.5	145.6 ± 0.4	0.8132	214.4 ± 0.4	0.806 ± 0.002
594.0	163 ± 2	0.7441	197 ± 2	0.740 ± 0.008
632.8	174 ± 3	0.6985	186 ± 3	0.699 ± 0.012

a function of α for different values of θ, whereas in figure 6 we plotted the measured intensity as a function of θ for different values of α. All these figures show good agreement between experimental and theoretical values, which verifies that the behaviour of the cellotape layer used is like that of a uniaxial crystal.

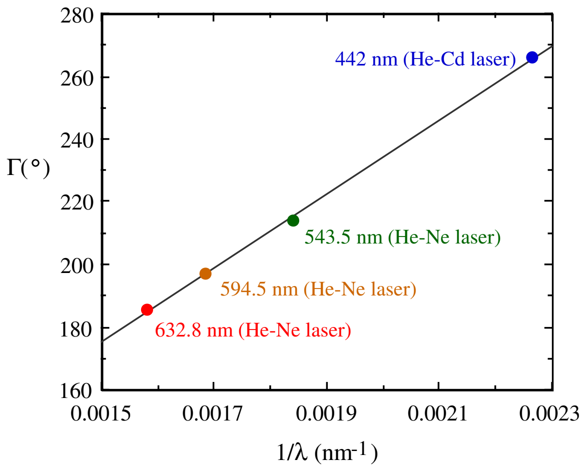

Figure 7. Retardation angle Γ as a function of $1/\lambda_0$ and linear fit.

Finally, in order to verify our hypothesis that Δn does not depend on the wavelength λ_0, we plotted Γ as a function of λ_0^{-1}, taking into account equation (20). As can be seen in figure 7 there is practically a linear relation between them. From this figure, we can do a least-squares fit to the experimental data and obtain an experimental value for Δnd for a single layer of cellotape, which is 326 ± 2 nm. Finally, we obtained the thickness d of the cellotape measuring the thickness of 16 layers and found that $16d = 0.68 \pm 0.02$ mm and then $d = 42.5 \pm 1.3$ μm, which allows us to conclude that the birefringence of the cellotape analysed is $\Delta n = 0.0077 \pm 0.0003$ for the interval of wavelengths considered. This result is only an approximation, since the difference between refractive indices, Δn, suffers, in general, from chromatic dispersion. However, the results in figure 7 suggest that there is no dispersion and we can, in fact, provide a single value for Δn. This can be a surprising result for the reader, but we can explain it considering now that Δn depends on the wavelength λ_0. A good approximation to the wavelength dependence of Δn is the Cauchy-type equation [7]

$$\Delta n = a + \frac{b}{\lambda_0^2} \tag{25}$$

where a and b are constant parameters. Then we can write equation (1) as follows:

$$\Gamma = \frac{2\pi}{\lambda_0}\Delta n\, d = \frac{A}{\lambda_0} + \frac{B}{\lambda_0^3}. \tag{26}$$

Our fit of the experimental data to equation (26) gives $A = 2036$ nm/rad and $B = 2.8326 \times 10^6$ nm^3/rad, and we obtain $a = 0.007\,624$ and $b = 10.61$ nm^2, which allows us to write equation (25) as follows:

$$\Delta n = 0.007\,624 + \frac{10.61}{\lambda_0^2}, \tag{27}$$

which takes values from 0.007 678 to 0.007 650 for wavelengths between 442 nm and 632.8 nm. These values can be approximated writing $\Delta n = 0.0077 \pm 0.0003$, which is the value obtained previously. Figure 8 shows the experimental values for the birefringence of a single cellotape layer and the theoretical fits obtained considering that Δn does not depend on the wavelength and that the wavelength dependence of Δn is the Cauchy-type equation.

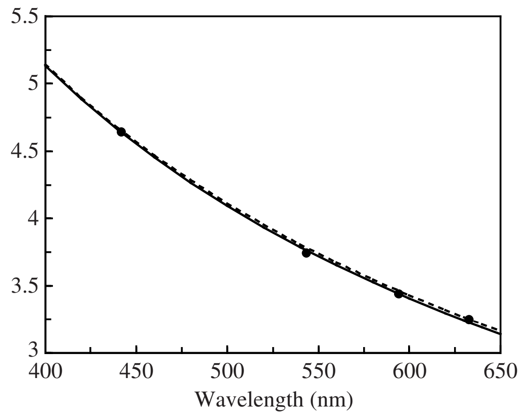

Figure 8. Birefringence versus the wavelength of a single cellotape layer. Experimental (dots), fitting considering that Δn is not a function of the wavelength (dashed line) and fitting considering that the wavelength dependence of Δn is the Cauchy-type equation (continuous line).

4. Conclusions

A simple system consisting of a strip of cellotape placed between two polarizers was used to analyse the behaviour of this cellotape layer as a birefringent retarder. The system was first theoretically analysed using Jones matrices, which allows us to easily obtain an expression for the intensity transmitted by the whole system as a function of the angles between the optic axis of the retarder and the transmission axis of the analyser, respectively, and the transmission axis of the polarizer. The use of monochromatic light of different wavelengths allowed us to obtain the true value of the retardation angle by measuring the transmitted intensity for an angle of 45° between the optic axis of the retarder and the transmission axis of the polarizer. Finally, the value of $\Delta n\, d$ for a single layer of cellotape is also obtained.

Acknowledgments

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