

# An experimental observation of Faraday's law of induction

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A small neodymium magnet moves with constant velocity through a coil, and the voltage induced is recorded with a computer interface. The observed voltage is compared to that predicted by simple calculations of magnetic flux using spherical polar coordinates. The close agreement between predicted and observed values combined with the experience gained in modeling the magnetic dipole field make this a very good experiment for the undergraduate student. © 2002 American Association of Physics Teachers.  
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## INTRODUCTION

A simple elegant experimental test of Faraday's law of induction can be done with equipment available in most physics teaching laboratories. While qualitative demonstrations of Faraday's law are commonly done in the physics classroom,<sup>1-4</sup> there have been few quantitative laboratory experiments in the area of electromagnetic induction that can be done with readily available apparatus.<sup>5</sup> Examples include magnets dropped through a sensing coil,<sup>6-8</sup> oscillated in a solenoid,<sup>9</sup> and moved at constant speed through a large coil.<sup>10</sup> Carpena<sup>11</sup> measured the induced voltage of a small magnet launched through a sensing coil to measure the speed of the magnet. In this article we describe an approach using a strong compact magnet moving with constant speed that is described by a model similar to that given by Carpena. This model predicts that the induced voltage extrema occur at half the coil radius above and below the coil and that the maximum induced voltage is proportional to the reciprocal of the square of the coil radius. Measurements of the location of the extrema and the values of induced voltage agree within a few percent with the model predictions, and experimental procedures are suitable for the introductory or advanced physics laboratory. The results provide a convincing experimental test of Faraday's law.

## EXPERIMENT

A small strong disk magnet moves with a constant velocity through a coil along the coil axis. The velocity of the magnet and the voltage induced in the coil are measured as functions of distance from the coil plane. Treating the magnet as an ideal dipole and the coil as having infinitesimally thin windings yields a simple model that predicts the induced voltage with an elementary calculation. Predicted and observed values agree closely for a coil with a radius more than twice that of the disk magnet. Significant deviations are observed with a coil having a radius 1.6 times the magnet radius.

The magnet used is a neodymium disk magnet 1.0 cm thick with a radius of 0.9 cm similar to that available from Master Magnets.<sup>12</sup> Three coils with 400 turns and average radii of 1.58, 2.26, and 2.83 cm were made by winding #32 gauge wire into rectangular slots milled into PVC pipe. The coil cross sections are about 1.0 cm wide and 0.3 cm thick. The magnet is supported on a balanced Atwood machine. The velocity of the magnet is measured by a Pasco rotary motion sensor and the induced voltage is recorded with a computer interface. The magnetic dipole moment of the

magnet is determined from on-axis measurements of the magnetic field made with a gaussmeter. The predicted and measured induced voltages are remarkably close, given the approximation of treating the magnet as an ideal dipole and the coil windings as being infinitesimally thin. The apparatus is shown in Fig. 1.

## MAGNETIC FLUX CALCULATION

Because of the small size of the magnet relative to the coil, the dipole approximation of the magnetic field is a suitable representation at distances from the magnet of three or more centimeters. The magnetic field of a dipole moment,  $m$ , oriented along the polar axis is given by<sup>13</sup>

$$\mathbf{B} = \frac{\mu_0 m}{4\pi r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta}). \quad (1)$$

The magnetic flux through the coil can be calculated, considering the magnet at the origin and using as the surface the spherical cap bounded by the coil as illustrated in Fig. 2. The magnetic flux through an area element of this cap is

$$\mathbf{B} \cdot d\mathbf{A} = \left( \frac{\mu_0 m}{4\pi r^3} 2 \cos \theta \right) r^2 \sin \theta d\theta d\phi. \quad (2)$$

Evaluation of the integral gives

$$\Phi = \int \mathbf{B} \cdot d\mathbf{A} = \frac{\mu_0 m N}{2r} \sin^2 \theta_0, \quad (3)$$

where  $\theta_0$  is the angle from the coil axis to the coil and  $N$  is the number of turns in the coil. Since  $\sin \theta_0 = a/r$  and  $r = (a^2 + z^2)^{1/2}$ , the flux as a function of  $z$  is

$$\Phi = \frac{\mu_0 m N}{2} \frac{a^2}{(a^2 + z^2)^{3/2}}. \quad (4)$$

## PREDICTION

From Faraday's law of induction the voltage generated as the magnet moves through the coil is given by

$$V = - \frac{d\Phi}{dt} = - \frac{d\Phi}{dz} \frac{dz}{dt} = - \frac{d\Phi}{dz} v \\ = \frac{3\mu_0 m N a^2}{2} \frac{zv}{(a^2 + z^2)^{5/2}}, \quad (5)$$

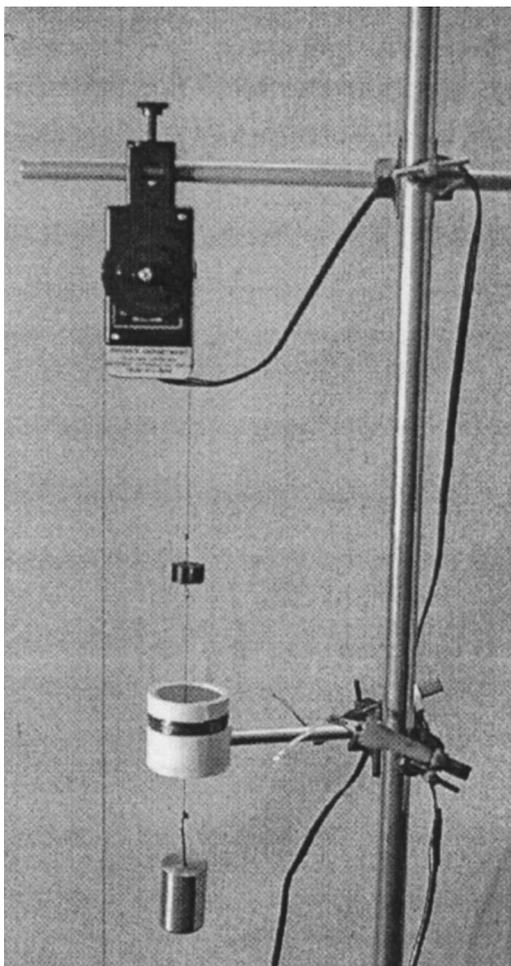


Fig. 1. The apparatus showing the rotary motion sensor, magnet, coil, and weight.

where  $v$  is the velocity of the magnet. This result predicts that the induced voltage is an antisymmetric function of the distance  $z$  from the coil plane. The derivative of  $V$  with respect to  $z$  is

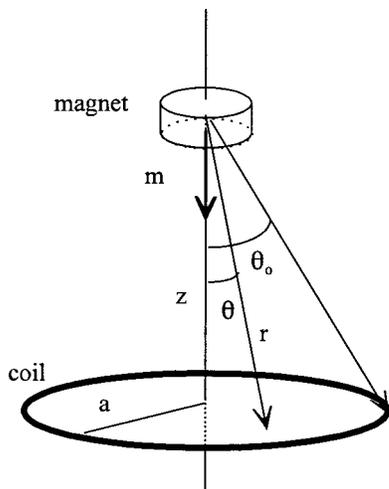


Fig. 2. Flux calculation for the magnet and coil.

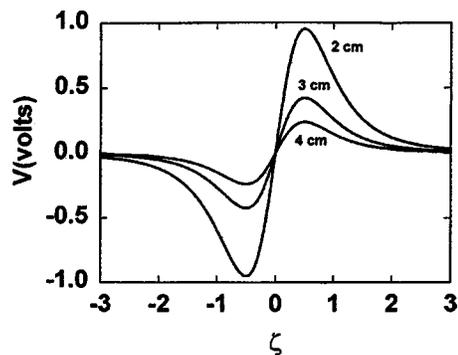


Fig. 3. Model plots of induced voltage vs distance from the coil plane in units of coil radius. The curves are for coils with 400 turns and radii of 2, 3, and 4 cm. Extreme values are proportional to  $1/a^2$  and occur at  $\pm 1/2$ .

$$V'(z) = \frac{3\mu_0 m N a^2 v}{2} \frac{(a^2 - 4z^2)}{(a^2 + z^2)^{7/2}} \quad (6)$$

so that the extreme values of  $V$  occur where  $V'=0$ , at  $z = \pm a/2$ . These values are

$$V_{e\pm} = \pm \frac{24\mu_0 m N v}{(5)^{5/2} a^2}. \quad (7)$$

This result indicates that the extreme values of the induced voltage are largest for coils with the smallest radius. Note that deviations from Eq. (7) are to be expected if the radius is sufficiently small to invalidate the ideal magnetic dipole and zero cross-sectional area approximations. We have obtained excellent agreement between observed and predicted voltages for a coil radius as small as 2.3 times that of the magnet. When the distance from the coil is measured in units of the coil radius,  $\zeta = z/a$ , the expression for the voltage in Eq. (5) becomes

$$V = \frac{3\mu_0 m N}{2a^2} \frac{\zeta v}{(1 + \zeta^2)^{5/2}}. \quad (8)$$

The plots in Fig. 3 show the variation of the induced voltage in terms of distance from the coil plane in units of  $z/a$  for three coil sizes, of radii 2, 3, and 4 cm. Note the antisymmetry of the plots, the location of the extrema at  $\pm 1/2$ , and the  $1/a^2$  dependence of the extreme values.

## MAGNETIC MOMENT DETERMINATION

From Eq. (1), the magnetic field of a dipole on axis is

$$B_z = \frac{\mu_0 m}{2\pi z^3}. \quad (9)$$

The magnetic field is measured on axis for distances from the magnet center of 3–10 cm. The slope of the graph of  $B$  vs  $1/z^3$  gives the value of the magnetic dipole moment from  $m = 2\pi \text{ slope} / \mu_0$ . A plot of these measurements given in Fig. 4 shows a close linear fit. The field 2 cm from the magnet fell 10% below this relation, indicating failure of the dipole approximation for distances less than 3 cm. The slope is  $4.70 \times 10^{-7} \text{ T m}^3$  yielding a value of  $2.35 \text{ A m}^2$  for the magnetic dipole moment.

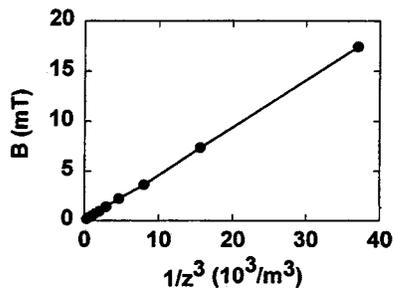


Fig. 4. A graph of the magnetic field of a disk magnet along its axis as a function of the reciprocal of the cube of the distance.

## MEASUREMENTS OF THE VELOCITY AND INDUCED VOLTAGE

The voltage and velocity measurements were made using a Pasco 750 Computer Interface with the large pulley of a Pasco rotary motion sensor. Data were sampled at 2500 Hz. The induced voltages detected with three coils are shown in Fig. 5. The mean radii of these coils are 2.83, 2.26, and 1.58 cm and the magnet speeds were 0.76, 0.73, and 0.85 m/s, respectively. The observed values of the voltages (shown by dots) are very close to the predicted ones (smooth curves). The radii used in the model curves were obtained by averaging the inside and outside radii of the coils. The magnet velocities were determined from the slope of the position versus time measurements provided by the rotary motion sensor. The location and values of the extremes of the voltage were determined by fitting a parabola to the regions near the extreme points. For each coil, data were recorded for five passages of the magnet through the coil. The average values of the locations of the extreme values are given in Table I, with  $z_{e-}$  the locations of the negative extreme values and  $z_{e+}$  those for the positive ones. The first column in Table I gives the value of half the coil radius,  $a/2$ , which is the location predicted by the model. Since the speed of the magnet was different for each run, the observed extreme voltages are divided by the speeds, providing a ratio that has a characteristic value for each coil. The average measured values for five runs are given in Table II. The first column gives the value predicted by Eq. (7) divided by the magnet speed. The indicated uncertainties in the last two columns in Tables I and II are the values of the standard deviation of results from the five runs. The uncertainties in the first column of Table I

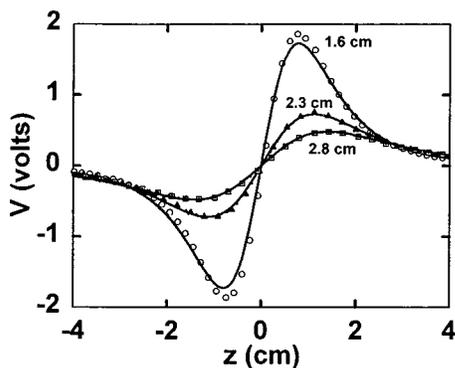


Fig. 5. Comparison of experimental and model voltages vs  $z$ . Data for coil radii of 1.6, 2.3, and 2.8 cm are indicated by circles, triangles, and boxes. The model curves for each case are shown as solid lines.

Table I. Comparison of predicted and observed locations of extreme voltage values.

Calculated $a/2$ (cm)	Measured	
	$-z_{e-}$ (cm)	$z_{e+}$ (cm)
$1.42 \pm 0.01$	$1.39 \pm 0.01$	$1.41 \pm 0.01$
$1.13 \pm 0.01$	$1.08 \pm 0.02$	$1.11 \pm 0.001$
$0.79 \pm 0.01$	$0.76 \pm 0.01$	$0.77 \pm 0.01$

result from an estimated radius measurement error of 0.2 mm. Uncertainty in the magnetic moment from the slope determination is 0.4% and the uncertainty from measurement of the magnetic field is about 1%. Combining these two with that from the radius yields total uncertainties for the predicted  $V/v$  values of 1.8% for the large coil, 2.1% for the middle coil, and 2.8% for the small coil. These uncertainties are indicated in the first column of Table II.

## RESULTS

The extreme values for the graphs occur at positions that are within a few percent of the predicted positions of  $\pm a/2$ . The differences between the observed and predicted extreme values are 1% for the largest coil with radius 2.83 cm, 2% for the middle-sized coil with radius 2.23 cm, and 9% for the smallest coil with radius of 1.58 cm. Only in the case of the small coil are the differences greater than the error range. The observed and predicted voltage curves are nearly identical except for the smallest one where significant differences are visible near the extreme values. Results from the smallest coil indicate that the ideal dipole and zero cross-sectional area assumptions are beginning to break down at this radius.

## CONCLUSION

This experiment provides students the opportunity to model the magnetic flux and induced voltage and to obtain results that agree remarkably well with the predictions of the model. They see that the model of an ideal magnetic dipole moving through a coil of wire that is infinitesimally thin predicts results that agree with the observed voltage measurements, and they can observe where those approximations begin to break down. They are able to observe the induced electromotive force which agrees with that predicted by Faraday's law of induction. The equipment needed for this experiment is present in most college physics laboratories or is inexpensive and available. Using a strong small magnet and measuring the velocity directly allows simple modeling and yields much better results than previously obtained.

Table II. Comparison of predicted and observed extreme voltage divided by speed values,  $V/v$ .

$ V/v_{epred} $ (V s/m)	$-V/v_{e-}$ (V s/m)	$V/v_{e+}$ (V s/m)
$0.632 \pm 0.011$	$0.629 \pm 0.001$	$0.625 \pm 0.005$
$0.994 \pm 0.021$	$1.016 \pm 0.010$	$1.025 \pm 0.010$
$2.026 \pm 0.057$	$2.210 \pm 0.020$	$2.190 \pm 0.009$

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## EXTENDING HUMAN EXPERIENCE

The theory still resists every attempt to visualize it, for ordinary vision turns out to be inadequate to grasp fundamental realities on the atomic scale. As Niels Bohr argued, our words and concepts are geared to our experiences as macroscopic beings, composed of vast numbers of atoms. However high it flies, our vaunted imagination still relies on its starting point, common human experience. To go where intuition fails, physics relies on logic and on the abstract language of mathematics, which tries to extend the speech of the tribe to a realm beyond the all-too-human.

Peter Pesic, *Seeing Double: Shared Identities in Physics, Philosophy, and Literature* (The MIT Press, Cambridge, MA, 2002), p. 97.