

Apparatus for Teaching Physics

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Measurement and Analysis of the Field of Disk Magnets

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Accurate measurements of millitesla (mT) and larger magnetic fields¹ allow one to study many aspects of magnetism associated with small magnets or coils. Solid-state devices for precision magnetic field measurement have become less expensive than in the past, are simple to use, and readily allow measurement of mT fields. Most² of these solid-state sensors are based on the Hall effect and are commercially used in automotive and industrial electronics. A very easy-to-use example is the Allegro Microsystems 3515 Linear Hall-Effect Sensor,³ which costs about \$2, gives direct voltage output with no external parts, and is accurate enough for introductory lab work. We discuss here the use of this chip

to make a simple magnetic probe that operates from a 9-V battery and may be used with a low-cost digital multimeter for magnetic field measurement.

The probe in total costs about \$5, and a suitable multimeter about \$20. For about \$25, quantitative measurements of commonly encountered magnetic fields can be easily made. With a precision voltmeter or by adding an amplifier to the probe, extremely accurate field measurements could be made using the sensor described here. This simple probe can be used to make measurements that demonstrate the accuracy of formulas for the field of disk magnets and give some insight into the strengths of permanent magnetic materials.

The Magnetic Probe

The 3515 chip looks like a plastic transistor with three leads and is about 4 x 3 x 1 mm, with a very small active area so that it measures very localized fields. It has ground, supply, and output pins, and needs a power supply voltage of 5 V. It is a low-power part easily run from a battery, requiring about 10 mA of current. In our circuit a 9-V battery is used, and a 78L05 voltage regulator places the correct voltage on the 3515's power pin. The sensitivity is 50 mV/mT with $\pm 10\%$ tolerance specified by the manufacturer. For precision measurement, a calibration could be done, but for many purposes this accuracy is adequate. In zero field, the device nominally gives 2.5 V, but the parts we have tested have averaged about 2.6 V. A magnetic field pointing into the device from above will decrease the voltage, while one from below will increase it. Thus, the direction of a field can be measured.

The field component in mT, along a line perpendicular to the sensor and into it from below, is given by $B = (V - V_0)/S$, where V is measured voltage, V_0 the zero field voltage, and S the sensitivity in V/mT. Using $S = 0.05$ will general-



Fig. 1. Magnetic probe is built using inexpensive parts and scraps.

(Photo: **Blaise MacMullin**)

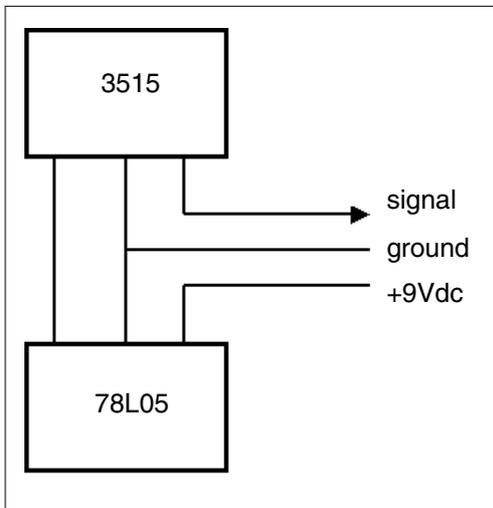


Fig. 2. At left, connections for the magnetic probe. The 78L05 regulator is glued with the flat side down on the plastic mounting material, and the 3515 is glued with lettering upward. The pins in contact when lined up this way are soldered together. The 9-V input is connected to the positive terminal of the battery through a push-button switch. The signal ground is connected to the negative terminal of the battery.



Fig. 3. Probe may be held in one hand and used to measure the magnetic field near a magnet. An inexpensive multimeter provides voltage readout.

(Photo: Blaise MacMullin)

ly suffice if V is in volts, while V_0 can be determined by using the probe away from field sources and pointed with the sensor normal horizontal and perpendicular to local magnetic north. According to testing in the manufacturer's laboratories,³ the device is 99.9% linear (100% being perfect linearity) in its response in the range -0.08 to 0.08 T. Tests at the Institute of Geophysics and Planetary Physics,

it for the duration of the course, it is essential that we reduce the cost of the instruments in those kits. This probe is the most inexpensive magnetic probe we have designed, and it is suitable even for young students using common magnets, although the following examples would be suitable for senior labs. As shown in Fig. 1, we glued a voltage regulator and magnetic sensor to a piece of scrap plastic, ran leads

for a multimeter off one end, and taped a 9-V battery on the bottom and a push-button switch on the top. Some holes were drilled in the plastic to allow wires to pass through for a neat layout. The chips and wiring were soldered together as in Fig. 2, and the exposed wires covered with model-airplane cement. The sensor may be plugged into a multimeter and held in one hand with the thumb activating the switch (see Fig. 3). We now proceed to see how this simple sensor can be used for meaningful quantitative measurements.

Quantitative Investigation of Magnetic Fields

An analytical theory exists for bar or disk magnets, and measured data can be plotted to check agreement with the theory. Further, a program such as Vernier Software's Graphical Analysis⁴ can be used to make an optimal fit of the theoretical curve to the data by adjusting parameters of the theoretical expression. Intermediate texts⁵ show that the magnetic field at an on-axis point at a distance x from the center of a disk-shaped magnet is similar to that of a solenoid and may be expressed by the integral

$$B(x) = \frac{\mu_0}{2} \int_{x-l/2}^{x+l/2} \frac{a^2 M}{(a^2 + z^2)^{3/2}} dz,$$

where the disk has thickness l , radius a , magnetization M , and μ_0 is the permeability of free space with value $4\pi \times 10^{-7}$ N/A².

Assuming M to be constant within the magnet, this may be integrated to yield:

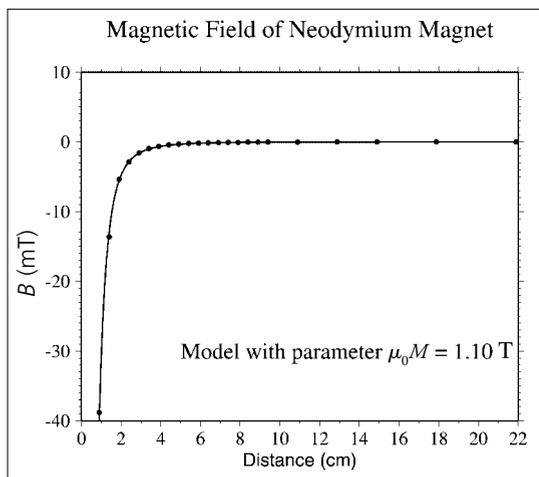


Fig. 4. Variation of magnetic field with distance from the center of a small Nd disk magnet. The line shows the theoretical curve best fit with the M parameter determined by Vernier's Graphical Analysis.

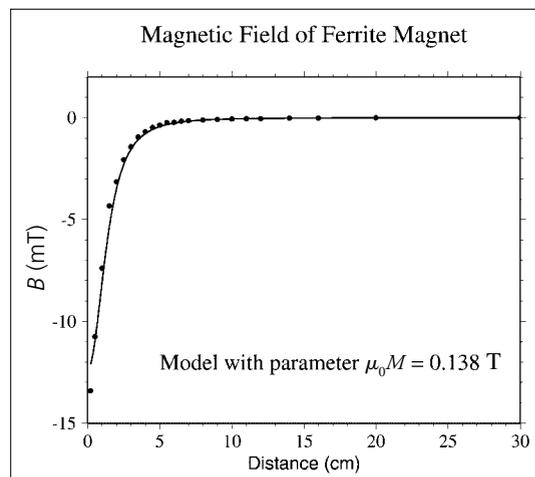


Fig. 5. Variation of magnetic field with distance from the center of a flattened ferrite disk magnet. The line shows the theoretical curve best fit with the M parameter determined by Vernier's Graphical Analysis.

$$B(x) = \frac{\mu_0 M}{2} \left[\frac{x + l/2}{\sqrt{(x + l/2)^2 + a^2}} - \frac{x - l/2}{\sqrt{(x - l/2)^2 + a^2}} \right].$$

The part of this expression in parentheses is purely geometric, and if the dimensions of a disk magnet are known, this function has in fact only one free parameter, which is M , a property of the magnetic material. Since the dependence on M is linear, simpler fitting methods could be used, but Graphical Analysis is useful for initial plotting of the data and then is well-suited to determining the single-fit parameter. A “neodymium” (Nd) magnet⁶ in the form of a disk 3.00 mm thick and 10.00 mm in diameter, with magnetic field perpendicular to its flat faces, was taped to a ruler and the on-axis field measured with the sensor at various distances. A four-digit multimeter was used to improve measurement accuracy. The data from the Nd magnet are shown in Fig. 4, with the best fit of the $B(x)$

function obtained by Graphical Analysis through variation of the value of M , with l and a fixed to the measured values of that magnet. An excellent fit has been obtained and the value of $\mu_0 M$, deduced from the fit, is 1.10 T. Magnet manufacturers typically give the value of “remanent magnetization” or “remanence” B_r , the value of B in a hysteresis loop when the magnetizing external field H has been removed. Since $B = \mu_0 (H + M)$, in this case of $H = 0$, B_r is equal to $\mu_0 M$. For comparison with listed magnetic properties of materials, it is thus more useful to use $\mu_0 M$ than the actual value of the magnetization M . For various Nd magnet alloys, Hitachi Metals⁷ gives values of B_r from 0.98 to 1.43 T. The measured value of $\mu_0 M$ is consistent with the disk magnet being made of

one of the lower-strength Nd alloys.

Common toy ferrite magnets are made from pressed metal powders and often have a nonmetallic appearance. To further test the theoretical fit, a flattened toy ferrite magnet was used. This magnet was only 3.10 mm in thickness but had a diameter of 34.6 mm and was taped to a ruler to have its field measured as before. As shown by Fig. 5, the $B(x)$ function presented above, with the geometric parameters of the magnet used, presents an extremely good fit to the data once the $\mu_0 M$ value has been optimized by Graphical Analysis. In this case the value corresponding to the remanence is 0.14 T. This is somewhat lower than the range of 0.22 to 0.4 T presented as typical of high-quality iron-barium or iron-strontium ferrites by a magnet vendor,⁸ although another vendor cites 0.115 to 0.14 T for Mn-Zn ferrites.⁹ The remanence value found seems reasonable given the fairly

large range apparently present in the various types of ferrite material. The model curve fits well for the majority of data points and is clearly useful both in representing the data and allowing a believable value of the remanent magnetization $\mu_0 M$ to be determined.

Comments

It has been shown that a simple unamplified sensor based on the Allegro 3515 chip can be used for simple yet meaningful quantitative experiments using common magnets. Interestingly, it was not possible with the unamplified sensor to detect magnetic fields far enough from the magnets in question that the simple power law with -3 exponent expected of a dipole could be detected. The fairly complex general equation for the on-axis field of a cylindrical magnet had to be used to represent the measured fields. With this simple sensor, the varying curves of the along-axis magnetic fields could be studied and well modeled using the physical theory for disk magnets of varying geometry and remanence. Other uses such as mapping fields around magnets readily suggest themselves, and imaginative scientific thought should produce many more uses for this sensor and others that could be easily designed based on the Allegro chip.

Acknowledgments

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References

1. Strictly speaking, one should refer to this as the magnetic induction or flux density, whose units are tesla (T) or webers per square meter (Wb/m^2) and customarily denoted B . The magnetic field intensity is usually denoted H and is measured in amperes per meter (A/m). In free space, the two are directly related. Earth's field is roughly 0.06 mT, below the useful range of the simple probe described here.
2. An alternative, the giant magnetoresistance sensor, was described by Joseph Priest, "Hands-on magnetic field measurements with a GMR sensor," *Phys. Teach.* 37, 345–347 (Sept. 1999).
3. Details at <http://www.allegromicro.com>, including distributor and sampling information. The exact part number is A3515LUA. The data sheet is available online, as are testing results described in Technical Paper STP97-10. A similar product from Honeywell is described in Shawn Carlson, "Detecting micron-size movements," *Sci. Am.* 275, 96 (August 1996); available at <http://www.sciam.com/0896issue/0896amsci.html>. An innovative use for magnets and sensors is shown in this article.
4. See <http://www.vernier.com>.
5. See, for example, H.E. Duckworth, *Electricity and Magnetism* (Holt, Rinehart, and Winston, New York, 1960), pp. 204–205, and P. Lorrain and D.R. Corson, *Electromagnetic Fields and Waves*, 2nd ed. (Freeman, San Francisco, 1970), pp. 317–381 and 396–405.
6. Common neodymium magnets are an alloy of neodymium, iron, and boron (NdFeB), and are the strongest commonly available magnets.
7. See <http://www.hitachimetals.com/products/mag/neodymium/properties.html>.
8. See <http://www.magnetsales.co.uk/>.
9. See <http://www.oemagnet.com/product/sf.htm>.