Second Collaborative homework

Note: Deadline is 15 March 2013, 10 am. Place your homework in the orange box on physics floor. Mention names of all collaborators in the group. Work alone or in groups up to five. Solution will be posted online at 10 am.

1. An electron in chromium makes a transition from the $n = 2$ state to the $n = 1$ state without emitting a photon. Instead, the excess energy is transferred to an outer electron (in the $n = 4$ state), which is ejected by the atom. (This is called an Auger process, and the ejected electron is referred to as an Auger electron.) Use the Bohr theory to find the kinetic energy of the Auger electron.

2. An electron initially in the $n = 3$ state of a one-electron atom of mass $M$ at rest undergoes a transition to the $n = 1$ ground state.
   (a) Show that the recoil speed of the atom from emission of a photon is given approximately by,
   \[ v = \frac{8hR}{9M}. \]
   (b) Calculate the percent of the $3 \rightarrow 1$ transition energy that is carried off by the recoiling atom if the atom is deuterium.
3. Apply classical mechanics to an electron in a stationary state of hydrogen to show that \( L^2 = m_e k e^2 r \) and \( L^3 = m_e k^2 e^4 / \omega \). Here \( k \) is the Coulomb constant, \( L \) is the magnitude of the orbital angular momentum of the electron, and \( m_e, e, r, \) and \( \omega \) are the mass, charge, orbit radius, and orbital angular frequency of the electron, respectively.

4. Wavelengths of spectral lines depend to some extent on the nuclear mass. This occurs because the nucleus is not an infinitely heavy stationary mass and both the electron and nucleus actually revolve around their common center of mass. It can be shown that a system of this type is entirely equivalent to a single particle of reduced mass \( \mu \) that revolves around the position of the heavier particle at a distance equal to the electron-nucleus separation. See Figure below.

Here, \( \mu = m_e M / (m_e + M) \), where \( m_e \) is the electron mass and \( M \) is the nuclear mass. To take the moving nucleus into account in the Bohr theory we replace \( m_e \) with \( \mu \).

Thus equation for allowed energy levels becomes,

\[
E_n = -\frac{\mu k e^2}{2 m_e a_0} \left( \frac{1}{n^2} \right),
\]

and equation for emitted wavelength becomes,

\[
\frac{1}{\lambda} = -\frac{\mu k e^2}{2 m_e a_0 h c} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = \left( \frac{\mu}{m_e} \right) R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right).
\]

Determine the corrected values of wavelength for the first Balmer line (\( n = 3 \) to \( n = 2 \) transition) taking nuclear motion into account for (a) hydrogen, \(^1H\), (b) deuterium,
\( ^2H \), and (c) tritium, \( ^3H \). (Deuterium, was actually discovered in 1932 by Harold Urey, who measured the small wavelength difference between \( ^1H \) and \( ^2H \).)

5. An electron with kinetic energy less than 100 eV collides head-on in an elastic collision with a massive mercury atom at rest.

(a) If the electron reverses direction in the collision (like a ball hitting a wall), show that the electron loses only a tiny fraction of its initial kinetic energy, given by,

\[
\frac{\Delta K}{K} = \frac{4M}{m_e(1 + M/m_e)^2},
\]

where \( m_e \) is the electron mass and \( M \) is the mercury atom mass.

(b) Using the accepted values for \( m_e \) and \( M \), show that,

\[
\frac{\Delta K}{K} = \frac{4m_e}{M},
\]

and calculate the numerical value of \( \Delta K/K \).