Quiz: Experimental Physics Lab-I

Name: **Roll no:**

Attempt all questions.

1. In an experiment, a ball of mass 100 g is dropped from a height of 65 cm into the sand container, the impact is called crater. Five students measured the diameter of the crater from the same height. The data for each student is shown in Figure (1), which student made the most precise measurement? [**3**]

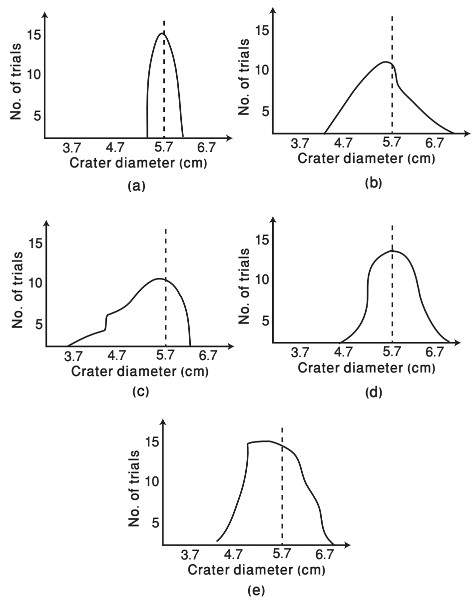

FIG. 1: Experimental data for crater formation.

Solution:

The correct answer is (**a**).

Since type-A uncertainty is evaluated statistically and can be minimized by repeating the experiment many times, therefore the spread of the Gaussian distribution associated with type-A uncertainties should be as thin as possible. Thus, the width of the crater diameter vs number of trials graph must have least spread. The only choice that shows this characteristic is choice (a).

2. The figure above represents a log-log (to the base 10) plot of variable y versus variable x. The origin represents the point $x = 1$ and $y = 1$. Which of the following gives the approximate functional relationship between y and x? [**3**]

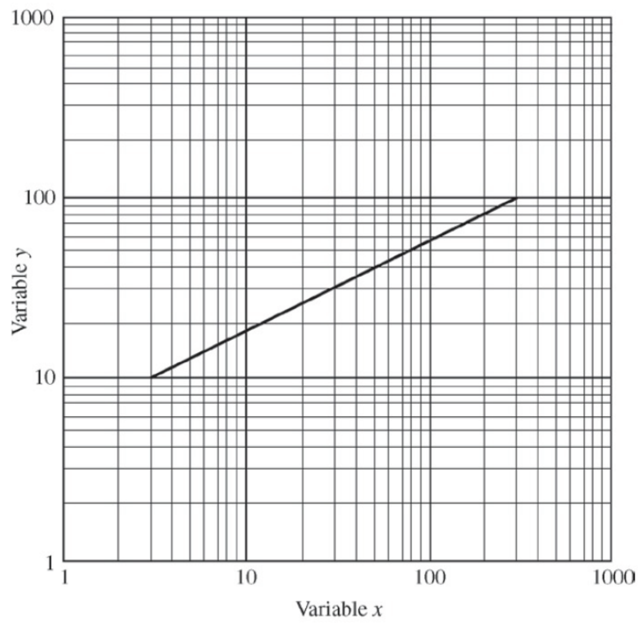

FIG. 2: Log-log plot of variable y versus variable x.

(a) $y = c\sqrt{x}$.

(b) $y = \frac{1}{2}x + c$.

(c) $y = 6x + c$.

(d) $y = cx^2$.

(e) $\log y = c + 5\log x$.

Solution:

The correct answer is (**a**).

A function of the form $y = cx^m$ will appear as a straight line on a log-log plot. Here m is the slope of the line and c is the $y-$value corresponding to $x = 1$. Therefore,

$$\begin{aligned} \log_{10}(y) &= \log_{10}(cx^m), \\ &= \log_{10}(c) + m\log_{10} x. \end{aligned}$$

The value of the slope can be found out,

$$m = \frac{\log_{10}(100) - \log_{10}(10)}{\log_{10}(300) - \log_{10}(3)},$$
$$= \frac{2-1}{2.477 - 0.477} = \frac{1}{2}.$$

The relationship between y and x will take the form,

$$\log_{10}(y) = \log_{10}(c) + \frac{1}{2}\log_{10} T, \qquad \text{or}$$
$$y = c\,x^{1/2}.$$

3. The volume V of a rectangular block is determined by measuring the length l_x, l_y and l_z of its sides. From the scatter of the measurements a standard uncertainty of 0.01% is assigned to each dimension. What is the fractional uncertainty in V, if,
(1) The scatter is due to uncertainties in setting and reading the measuring instrument.
(2) If it is due to temperature fluctuations? [**3**]

(a) 0.2% and 0.3% respectively.

(b) 0.02% and 5% respectively.

(c) 0.02% and 0.03% respectively.

(d) 0.03% and 0.02% respectively.

(e) None of the above

Solution:

The correct answer is (**c**).

(1) The standard uncertainty in each dimension is 0.01%. The volume of a rectangular block is,

$$V = l_x l_y l_z.$$

The uncertainty affects the three sides independently. Hence, the standard uncertainty in V can be calculated through the Taylor series approximation,

$$\Delta V = \sqrt{\left(\frac{\partial V}{\partial l_x}\Delta l_x\right)^2 + \left(\frac{\partial V}{\partial l_y}\Delta l_y\right)^2 + \left(\frac{\partial V}{\partial l_z}\Delta l_z\right)^2},$$
$$= \sqrt{\left(l_y l_z \Delta l_x\right)^2 + \left(l_x l_z \Delta l_y\right)^2 + \left(l_x l_y \Delta l_z\right)^2}.$$

Dividing both sides of the above expression by V yields,

$$\frac{\Delta V}{V} = \sqrt{\left(\frac{l_y l_z}{l_x l_y l_z}\Delta l_x\right)^2 + \left(\frac{l_x l_z}{l_x l_y l_z}\Delta l_y\right)^2 + \left(\frac{l_x l_y}{l_x l_y l_z}\Delta l_z\right)^2},$$

$$= \sqrt{\left(\frac{\Delta l_x}{l_x}\right)^2 + \left(\frac{\Delta l_y}{l_y}\right)^2 + \left(\frac{\Delta l_z}{l_z}\right)^2},$$

$$= \sqrt{(0.01)^2 + (0.01)^2 + (0.01)^2} = 0.017\%,$$

$$= 0.02\%.$$

(2) For temperature variations, all sides are affected equally. Therefore, one can use the formula for volume with equal lengths,

$$V = l^3,$$

and the uncertainty in V is,

$$\Delta V = \sqrt{\left(\frac{\partial V}{\partial l}\Delta l\right)^2} = 3l^2 \Delta l.$$

Dividing by V on both sides gives,

$$\frac{\Delta V}{V} = \frac{3l^2 \Delta l}{l^3} = 3\left(\frac{\Delta l}{l}\right), 3 \times 0.01\%,$$

$$= 0.03\%.$$

This result shows that the overall uncertainty can increase, if uncertainties are not independent nor random.

4. Figure (3A) shows the position $x(t)$ versus time plot for an elevator cab that is initially stationary, then moves upward (which we take to be the positive direction of x), and then stops. Choose the best option for the velocity $v(t)$ and acceleration $a(t)$ shown in Figure (3B). [**3**]

<u>**Solution:**</u>

The correct answer is (**d**).

The slope ($v = dx/dt$) of $x(t)$ is zero in the intervals from a to b and at point d, this means that the cab is stationary. During the interval bc, the slope is constant

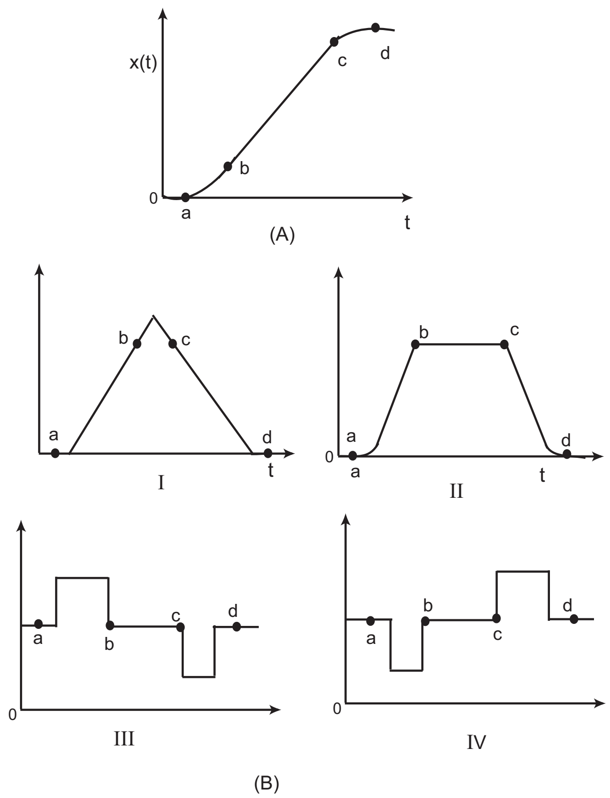

FIG. 3: (A) Position versus time graph, (B) graphs for velocity and acceleration.

	$v(t)$ vs t	$a(t)$ vs t
(a)	II	IV
(b)	I	II
(c)	I	IV
(d)	II	III
(e)	I	III

and nonzero and the cab moves with constant velocity indicated by bc in subfigure (II). Since the cab initially begins to move and then later slows to a stop, v varies as indicated by the slopes of $b - 0$ and $c - 0$ in the subfigure (II). Thus, subfigure (II) is the required plot.

The acceleration of a particle at any instant is the rate at which its velocity is changing at that instant. Graphically, the acceleration at any point is the slope of the curve of

$v(t)$ at that point, therefore,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}.$$

Comparing subfigure (III) with subfigure (II), each point on subfigure (III) shows the derivative (slope) of the $v(t)$ curve at the corresponding time. When v is constant, the derivative is zero and so also is the acceleration. When the cab first begins to move, the $v(t)$ curve has a positive derivative (the slope is positive), which means that $a(t)$ is positive. When the cab slows to stop, the derivative and slope of the $v(t)$ curve is negative, hence $a(t)$ is negative.

5. The period of oscillations T of a body constrained to rotate about a horizontal axis for small amplitudes is given by the expression,

$$T = 2\pi\left(\frac{I}{mgd}\right)^{1/2}, \tag{1}$$

where m is mass of the body , d is the distance between center of mass (CM) and the axis of rotation and I is the moment of inertia (MI) about the axis of rotation given by (from parallel axis theorem: $I = I_o + md^2$). Here I_o is the moment of inertia about parallel axis through center of mass . If k is the radius of gyration that depends on geometry i.e., $k = \frac{l^2+b^2}{12}$, then $I_0 = mk^2$.
Now Equation (1) can be written as,

$$T = 2\pi\sqrt{\frac{k^2+d^2}{gd}}. \tag{2}$$

How would you find the value of g by plotting? [**3**]

(a) T versus d.

(b) T^2d versus d^2.

(c) T^2 versus $\sqrt{d}$.

(d) T versus $\log(d)$.

(e) All of the above.

Solution:

The correct answer is (**b**).

By looking at Equation (2), one can tell that if we plot T versus d that will follow a parabola trend. The best way is to linearize the given function which is an important technique from data analysis perspective. Rearranging Equation (2) yields,

$$T^2 d = \left(\frac{4\pi^2}{g}\right) d^2 + \frac{4\pi^2 k^2}{g}.$$

This is a straight line function with $T^2 d$ as the dependent variable, d^2 as the independent variable, $(4\pi^2/g)$ is the slope while $(4\pi^2 k^2/g)$ is the intercept.
The g value can be computed through the resultant outcome of slope $(4\pi^2/g)$.

6. Lissajous figures are used for the measurement of phase and produced when one signal is connected to the vertical trace of the oscilloscope and the other to the horizontal trace. If the two signals have the same frequency, then the lissajous figure will assume the shape of an ellipse. The ellipse's shape varies according the phase difference between two signals and according to the ratio of amplitudes of the two signals. The phase difference can be calculated through the following expression,

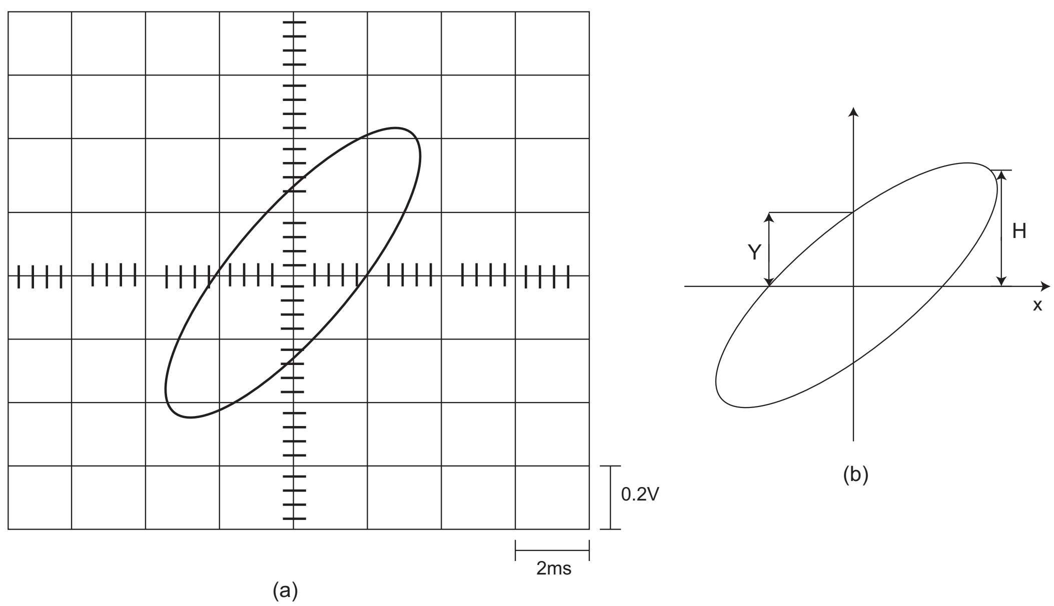

FIG. 4: (a) The output signal of an oscilloscope, and (b) an ellipse.

$$\sin(\phi) = \pm\frac{Y}{H}, \tag{3}$$

where H is half the maximum height and Y is the intercept on the $y-$axis as shown in Figure (4b).

Calculate the phase difference ϕ and its uncertainty (assume this is an analog scale) based on the Lissajous pattern given in Figure (4a). [**3**]
(Hint: $\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$).

Solution:

Since each block on the oscilloscope screen is equivalent to 0.2 V, therefore by reading the scale, the values of the ellipse's parameters becomes,

$$
\begin{aligned}
Y &= 0.2 + 0.05 = 0.25\,\text{V}, \\
X &= 0.4 + 0.04 = 0.44\,\text{V}.
\end{aligned}
$$

Substituting these values in Equation (3) yields,

$$
\begin{aligned}
\phi &= \sin^{-1}\left(\frac{Y}{H}\right), \\
&= \sin^{-1}\left(\frac{0.25}{0.44}\right) = 34.62^\circ, \\
&= 35^\circ.
\end{aligned}
$$

Since this is an analog scale, uncertainties associated with Y and H can be found out as,

$$
\begin{aligned}
u_{_Y} &= \frac{\Delta}{\sqrt{6}} = \frac{0.04/2}{\sqrt{6}} = 0.008\,\text{V}, \\
u_{_H} &= \frac{\Delta}{\sqrt{6}} = \frac{0.04/2}{\sqrt{6}} = 0.008\,\text{V}.
\end{aligned}
$$

The values of Y and H can be quoted as,

$$
\begin{aligned}
Y &= (0.250 \pm 0.008)\,\text{V}, \\
H &= (0.440 \pm 0.008)\,\text{V}.
\end{aligned}
$$

Notice that uncertainty has only one significant figure and the decimal places of both the original quantity and the uncertainty are at the same position.

The uncertainty in the phase ϕ can be calculated through the Taylor series approximation,

$$
\Delta\phi = \sqrt{\left(\frac{\partial\phi}{\partial Y}\Delta Y\right)^2 + \left(\frac{\partial\phi}{\partial H}\Delta H\right)^2}. \tag{4}
$$

Differentiating equation (3) w.r.t Y gives,

$$\frac{\partial\phi}{\partial Y} = \frac{1}{\sqrt{1-\left(\frac{Y^2}{H^2}\right)}}\left(\frac{d(Y/H)}{dY}\right),$$
$$= \frac{1}{\sqrt{1-\left(\frac{Y^2}{H^2}\right)}}\left(\frac{1}{H}\right),$$
$$= \frac{1}{\sqrt{1-\left(\frac{0.25^2}{0.44^2}\right)}}\left(\frac{1}{0.44}\right),$$
$$= 2.76.$$

Differentiating equation (3) w.r.t H yields,

$$\frac{\partial\phi}{\partial H} = \frac{1}{\sqrt{1-\left(\frac{Y^2}{H^2}\right)}}\left(\frac{d(Y/H)}{dH}\right),$$
$$= \frac{1}{\sqrt{1-\left(\frac{Y^2}{H^2}\right)}}\left(-\frac{Y}{H^2}\right),$$
$$= \frac{1}{\sqrt{1-\left(\frac{0.25^2}{0.44^2}\right)}}\left(\frac{0.25}{0.44^2}\right),$$
$$= 1.57.$$

Substituting in Equation (4) results in,

$$\Delta\phi = \sqrt{(2.76\times 0.008)^2 + (1.57\times 0.008)^2},$$
$$= 0.025 = 0.02.$$

The uncertainty value in degrees would be,

$$\Delta\phi = 0.02\times(\frac{180}{3.14}) = 1.1^\circ.$$

Hence, the final value of phase ϕ can be quoted as,

$$\phi = (35\pm 1)^\circ.$$

Formula sheet:

Taylor series approximation: If a quantity $q = q(x, y, z)$ is measured using some input variables x, y and z which are measured with uncertainties Δx, Δy and Δz, respectively, then Δq can also be find out using the Taylor series approximation given as,

$$\Delta q = \sqrt{\left(\frac{\partial q}{\partial x}\,\Delta x\right)^2 + \left(\frac{\partial q}{\partial y}\,\Delta y\right)^2 + \left(\frac{\partial q}{\partial z}\,\Delta z\right)^2}.$$

Standard deviation: $s = \sqrt{\frac{\sum_i d_i^2}{N}}$.

Standard uncertainty: $\sigma = \sqrt{\frac{N}{N-1}}\,(s)$.

Standard uncertainty in the mean: $\sigma_m = \frac{\sigma}{\sqrt{N}}$.

Weighted average: $x_{avg} = \frac{\sum w_i x_i}{\sum w_i}$

Slope (m) and intercept (c) with equal weights:

$$m = \frac{\sum_i^N y_i(x_i - \bar{x})}{\sum_i^N (x_i - \bar{x})^2} \quad \text{or} \quad m = \frac{N\sum_i^N x_i y_i - \sum_i^N x_i \sum_i^N y_i}{N\sum_i^N x_i^2 - (\sum_i^N x_i)^2} \tag{5}$$

$$c = \bar{y} - m\bar{x} \quad \text{or} \quad c = \frac{\sum_i^N x_i^2 \sum_i^N y_i - \sum_i^N x_i \sum_i^N x_i y_i}{N\sum_i^N x_i^2 - (\sum_i^N x_i)^2}. \tag{6}$$

Uncertainty in slope m and intercept c is given as,

$$u_m = \sqrt{\frac{\sum_i^N d_i^2}{D(N-2)}}, \tag{7}$$

$$u_c = \sqrt{\left(\frac{1}{N} + \frac{\bar{x}^2}{D}\right)\left(\frac{\sum_i^N d_i^2}{(N-2)}\right)}, \tag{8}$$

where,

$$d_i = y_i - m x_i - c,$$
$$D = \sum_i^N (x_i - \bar{x})^2.$$

Slope m and intercept c with unequal weights

The weights are reciprocal squares of the total uncertainty (u_{Total}),

$$w = \frac{1}{u^2_{\text{Total}}}. \tag{9}$$

The mathematical relationships for slope (m) and intercept (c) are,

$$m = \frac{\Sigma_i w_i \, \Sigma_i w_i (x_i y_i) - \Sigma_i (w_i x_i) \, \Sigma_i (w_i y_i)}{\Sigma_i w_i \, \Sigma_i (w x_i^2) - (\Sigma_i w_i x_i)^2}\,, \tag{10}$$

$$c = \frac{\Sigma_i (w_i x_i^2) \, \Sigma_i (w_i y_i) - \Sigma_i (w_i x_i) \, \Sigma_i (w_i x_i y_i)}{\Sigma_i w_i \, \Sigma_i (w_i x_i^2) - (\Sigma_i w_i x_i)^2}, \tag{11}$$

where x is the independent variable, y is the dependent variable and w is the weight.

The expressions for the uncertainties in m and c are,

$$u_m = \sqrt{\frac{\Sigma_i w_i}{\Sigma_i w_i \, \Sigma_i (w_i x_i^2) - (\Sigma_i w_i x_i)^2}}\,, \tag{12}$$

$$u_c = \sqrt{\frac{\Sigma_i (w_i x_i^2)}{\Sigma_i w_i \, \Sigma_i (w_i x_i^2) - (\Sigma_i w_i x_i)^2}}. \tag{13}$$