Final Exam 2013: Modern Physics

Name: 
Roll no:

- Write your name and roll number in the space specified above.
- This exam comprises two parts, A and B. Part A comprises 25 questions. The most appropriate answer is to be circled on the question paper. Part A is to be filled on the question paper and returned.
- For answering part B, you will use the provided blue answer books. There are two questions in part B.
- Part A contains 15 pages including this page.
- Part B contains 2 pages.
Fundamental constants and other useful information

\[ c = \text{Speed of light} = 3.0 \times 10^8 \text{ ms}^{-1} \]

\[ h = \text{Planck’s constant} = 6.67 \times 10^{-34} \text{ Js} \]

\[ \hbar = \text{Reduced Planck’s constant} = \frac{h}{2\pi} = 1.06 \times 10^{-34} \text{ Js} \]

\[ k_B = \text{Boltzmann constant} = 1.38 \times 10^{-23} \text{ J/K} \]

\[ R = \text{Rydberg constant} = 1.1 \times 10^7 \text{ m}^{-1} \]

\[ m_e = \text{Mass of electron} = 9.11 \times 10^{-31} \text{ kg} \]

\[ m_p = \text{Mass of proton} = 1.67 \times 10^{-27} \text{ kg} \]

\[ 1\text{eV} = 1.6 \times 10^{-19} \text{ J} \]

\[ g = \text{Acceleration due to gravity} = 10.0 \text{ m s}^{-2} \]

\[ G = \text{Gravitational constant} = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \]

\[ \frac{1}{4\pi\varepsilon_0} = \text{Coulomb’s constant} = 9 \times 10^9 \text{ N m}^2/\text{C}^2 \]

\[ \text{TISE : } -\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \]

\[ \text{TDSE : } -\frac{\hbar^2}{2m} \frac{d^2\Psi(x, t)}{dx^2} + V(x)\Psi(x, t) = i\hbar \frac{d}{dt}\Psi(x, t) \]
PART A

Attempt all questions. Mark your answers on these sheets and return. All MCQ’s are three marks each.

1. A metal is held at zero voltage. The energy diagram at the metal-air interface is shown.

\[ \text{Unfilled levels} \quad \phi \quad \text{Filled levels} \]

In thermionic emission, electrons are ejected from the metal surface because:

(a) The work function $\phi$ increases.

(b) The work function $\phi$ decreases.

(c) The potential energy seen by the electrons in the air slopes downward.

(d) Increasing temperature makes more electrons jump into unfilled levels increasing the fraction of electrons with thermal energy beyond $\phi$.

(e) The Fermi level $E_F$ decreases.
2.

The figure shows the energy diagram for a metal in which electrons fill energy levels up to $E_F$. A thin insulating oxide layer separates the metal from a quantum dot with only ten quantized energy levels. The quantum dot is given a positive potential $V_0$ with respect to the metal, enabling an electron to tunnel across the oxide layer. Which one of these plots shows the correct behavior of the tunneling current $i$ from metal to the quantum dot. At $V_0 = 0$, $E_F$ is at the same energy as the $n = 7$ quantum level.
3. An electron is injected into a potential energy landscape from the left region I as shown below. It encounters a potential step. The energy of the electron is $E$ and $E < |V_0|$. If the electron is to emerge in region III with a faster speed, the appropriate potential step is given by which of the following?

(a) $E - V_0$
(b) $E + V_0$
(c) $E + V_1$
(d) $E - V_1$
(e) The speed of the electron cannot increase.

4. A pendulum is a harmonic oscillator. It completes one round trip in 1 s. According to quantum physics, its minimum energy in Joules is,

(a) Zero.
(b) $6.63 \times 10^{-34}$.
(c) I need to know the angle $\theta$ to answer this question.
(d) $3.315 \times 10^{-34}$.
(e) It is negative.
5. Suppose the maximum angle $\theta$ of the pendulum discussed in the previous question is such that the bob goes to a maximum height of 1 mm. The mass of the pendulum is 1 gram and $g = 10 \text{ ms}^{-2}$. The bob starts oscillating and eventually comes to rest, losing all of its energy. Energy is lost in the form of a phonon when the oscillator makes a transition from the $p/th$ quantum level to $(p-1)^{th}$ quantum level. How many phonons are emitted in the process?

(a) Negligible.  
(b) One.  
(c) $\approx 1.5 \times 10^{28}$.  
(d) $\approx 1.5 \times 10^{33}$.  
(e) None of the above.

6. Snell’s law of refraction determines the bending of light across an interface. For sure, electrons are also waves and can be refracted. The corresponding law for electrons is called Bethe’s law and is given by,

$$\frac{\sin \alpha}{\sin \beta} = \frac{v_2}{v_1},$$

where $\alpha$ is the angle of incidence measured from the normal to the interface, $\beta$ is the angle of refraction also measured from the normal, $v_1$ is the speed of electron in the incident medium and $v_2$ is the speed in the refracted medium. Now a beam of electrons is made to pass through two hollow cylinders with an applied voltage difference. Which of the following diagrams show the correct trajectory of electrons?
7. A proton of rest mass $m_0$ is accelerated to half the speed of light. Its de Broglie wavelength is,

(a) $\frac{2h}{m_0c}$.  
(b) $\frac{2h}{\sqrt{3}m_0c}$.  
(c) $\frac{4h}{\sqrt{3}m_0c}$.  
(d) $\frac{4h}{m_0c}$.  
(e) A proton does not have a wavelength.

8. An electron starts off in the region $B$, trapped in a well. The potential energy $V(x)$ along position $x$ is shown.
Now suppose some time lapses. From a quantum viewpoint, which of the regions A or C, is the electron more probable to be found?

(a) Region A.

(b) Region C.

(c) Equal probability of being found in A and C.

(d) The electron absolutely cannot leave region B.

(e) None of the above.

9. A particle moving in a region of zero force encounters a precipice—a sudden drop in the potential energy to an extremely large negative value. What is the probability that it will “go over the edge”, i.e., it will enter the negative potential energy region?

(a) Almost zero.  
(b) Almost one.

(c) \( \approx 1/2 \).  
(d) \( > 1/2 \).

(e) None of the above.

10. An LED (light emitting diode) emits light when electrons fall from a top set of levels (conduction band) to a bottom set of levels (called a valence band). These levels are separated by an energy gap \( E_g \). An electric current supplies the electrons.
A current of 2.5 mA flows through an LED with $E_g = 1.4$ eV. Assuming that each current-carrying electron drops into a hole, thereby emitting a single photon, what is the power emitted in the light?

(a) $5.6 \times 10^{-22}$ W. (b) 3.5 mW. (c) 2.5 mW. (d) 1.4 mW. (e) The power cannot be calculated using the provided information.

11. An electron is trapped inside a three-dimensional quantum dot. The energy is quantized in three dimensions according to,

$$E_{n_x,n_y,n_z} = \frac{\pi^2 \hbar^2}{2m} \left( \frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2} \right),$$

where $a$, $b$ and $c$ are the confining dimensions of the box (= dot) and $n_x$, $n_y$, $n_z$ are the three quantum numbers, each one of them being a positive integer.

If $a = b = c$, the energy difference between the ground and the first excited state is,

(a) $\frac{\pi^2 \hbar^2}{2ma^2}$. (b) $9 \frac{\pi^2 \hbar^2}{2ma^2}$. (c) $3 \frac{\pi^2 \hbar^2}{2ma^2}$. (d) $\frac{\pi^2 \hbar^2}{ma^2}$. (e) There are more than one “first excited states” all with different energies. Hence this question cannot be answered.

12. If a laser pulse is of a short enough duration, it becomes meaningless to refer to its specific color. How short a duration must a light pulse be for its range of frequencies to cover the entire visible spectrum, and hence produce what is called a “white laser”?

The visible spectrum covers frequencies of $\sim 4.5 \times 10^{14}$ to $7.5 \times 10^{14}$ Hz.
13. A free particle has a wave function,

\[ \Psi(x, t) = Ae^{i(2.5 \times 10^{11} x - 2.1 \times 10^{13} t)}, \]

where \( x \) is in metres and \( t \) is in seconds. What is the mass of the particle?

(a) Mass can only be determined if \( A \) is known.
(b) 0.012 kg.
(c) 0.11 kg.
(d) 5.7 \times 10^{-16} \text{ kg}.
(e) 1.7 \times 10^{15} \text{ kg}.

14. A magic-eye was shown as a classroom demonstration. In this gadget, a beam of electrons is emitted from a cathode and moves horizontally towards a bowl shaped anode. When viewed from top, the electrons produced a characteristics glow as shown.

If the tube is placed inside a perpendicular magnetic field pointing out of the plane of the paper, as shown in Fig (b), the view from the top will look like:
15. A beam of electrons accelerated through a potential difference \( V_0 \) is directed at a single slit of width \( a \), then detected at a screen at a distance \( L \) beyond the slit. How far from a point directly in the line of the beam is the first location where no electrons are ever detected?

(a) \( L/(a\sqrt{V}) \).  
(b) \( Lh/(a\sqrt{2meV}) \).

(c) The electrons are detectable everywhere.

(d) \( L/a \).  
(e) \( Lh/(2a\sqrt{2meV}) \).

16. An electron of energy 1 eV is trapped inside an infinite well of length 30 cm. What is the distance between two consecutive nodes of the electron’s wavefunction? (A node is a point where the wavefunction goes to zero.)

(a) There are no nodes in the electron’s wavefunction.

(b) The distance between consecutive nodes is zero.

(c) \( 1.25 \times 10^{-18} \) m.

(d) \( 6 \times 10^{-10} \) m.

(e) None of the above.

17. For the electron in the previous question, will quantum effects be visible? Please give a reason.  [2 Marks]

Answer:
18. An electron is trapped in an infinite well of length $L$ and ground state energy $E_1$. At $t = 0$, the wavefunction is,

$$\Psi(x, 0) = \frac{1}{\sqrt{5L}} (\psi_1(x) + 2\psi_2(x)),$$

where $\psi_1(x)$ and $\psi_2(x)$ are normalized wavefunctions in the ground and first excited states. The wavefunction at $t = \pi \hbar / E_1$ is given by:

19. The potential energy profile in a certain region is shown.

Date: 16 May, 2013
A particle of energy $E$ exists inside this region. A sketch of the possible (real part) of the wavefunction is;

(a) 
(b) 
(c) 
(d) 
(e) None of the above.

20. Suppose a particle is in the ground state with wavefunction $\psi_1(x)$. Which one of the following is the probability that the particle will be found in a narrow range between $x$ and $x + dx$.

(a) $|\psi_1(x)|^2 dx$.  
(b) $x|\psi_1(x)|^2 dx$.  
(c) $\int_{x}^{x+dx} x|\psi_1(x)|^2 dx$.  
(d) $\int_{-\infty}^{+\infty} x|\psi_1(x)|^2 dx$.  
(e) None of the above.

21. At time $t = 0$, the state for a particle inside an infinite well is $\frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$, where $\psi_1$ and $\psi_2$ are ground and first excited states: with energies $E_1$ and $E_2$ respectively.

We first measure the position of the particle at time $t = 0$ and obtain the result $x_0$. 

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Immediately after the position measurement, we measure the energy. What possible result(s) can we obtain for the energy measurement?

(a) We can only measure either $E_1$ or $E_2$.
(b) We can obtain one of the energy values $E_n = n^2 \frac{x^2 k^2}{2ma}$, where $n$ can be an arbitrary large integer.
(c) We can only measure $\frac{1}{2}(E_1 + E_2)$.
(d) We may measure any energy $E = \sum_{n=1}^{\infty} C_n E_n$, where $C_n$ are coefficients so that $\sum_{n=1}^{\infty} |C_n|^2 = 1$.
(e) None of the above.

22. A free particle has a wavefunction $A(e^{ikx} + e^{-ikx})$ and energy $E$. $A$ is a normalization constant. Mark True or False against these statements.

(i) The probability density does not change with time.
(ii) The probability density is constant in space $x$.
(iii) The de Broglie wave associated with the particle is in fact a standing wave.

[6 Marks]

23. The uncertainty relationship for a particle moving in a straight line is $\Delta p \Delta x \geq \hbar/2$.

If the particle is moving in a circle with angular momentum $L$, the uncertainty relationship becomes:

(HINT: Distance becomes the arc length!)

(a) $\Delta L \Delta \theta \geq \frac{\hbar}{2}$. 

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(b) $\Delta L \Delta S \geq \frac{\hbar}{2}$.
(c) $\Delta L \Delta R \geq \frac{\hbar}{2}$.
(d) $\Delta L \Delta \theta \leq \frac{\hbar}{2}$.
(e) None of the above.

24. In a scanning tunneling microscope (STM), the tunneling probability of electrons from metal surface to a prob tip is proportional to $\exp(-2\alpha L)$, where $L$ is the tip-sample distance and $\alpha = 1 \text{ nm}^{-1}$ is the inverse of the penetration length.

If the tip moves closer to the surface by $\Delta L = 0.1 \text{ nm}$, the tunneling current,

(a) remains unchanged.
(b) increase by 22 %.
(c) decrease by 22 %.
(d) increase by 10 %.
(e) decrease by 10 %.
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PART B

Attempt all questions.

1. The Heisenberg uncertainty principle applies to photons as well as to material particles. Thus a photon confined to a small box of size $\Delta x$ necessarily has a large uncertainty in momentum and uncertainty in energy. Recall that for a photon $E = pc$.

(a) Estimate the uncertainty in energy for a photon confined to the tiny box of size $\Delta x$. [2 Marks]

(b) If $\Delta E \sim E$, what is the effective mass of the photon? [2 Marks]

(c) This mass can be extremely large, if $\Delta x$ is tiny. If $\Delta x$ is sufficiently small, the large mass can create a large gravitational field, sufficiently large to form a black hole. When this happens $\Delta x$ is called the Planck length, and this is when gravity and quantum mechanics become inter mixed. For a black hole, not even light can escape.

Consider a star of mass $M$ and radius $R$ as shown above. If an object is to be launched from the star’s surface so that it escapes the star’s
gravitational pull, it needs a minimum velocity \( v_{esc} \) called the escape velocity. Show that \( v_{esc} = \sqrt{\frac{2GM}{R}} \). [4 Marks]

(d) If \( v_{esc} = c \), nothing can escape from this star, not even light. If we were to replace the star of mass \( M \) with a photon of the mass calculated in part (b), and confined to length \( \Delta x \), and set \( R = \Delta x \), calculate the Planck length in terms of \( G \), \( h \) and \( c \). [3 Marks]

(e) If \( G = 6.67 \times 10^{-11} \) N m\(^2\)/kg\(^2\), find the numerical value of Planck’s length. [2 Marks]

(f) What is the diameter of a proton (about 2 fm = 2 \( \times \) \( 10^{-15} \) m) in units of Planck’s length? [2 Marks]

2. The radioactive decay of certain heavy nuclei by emission of an alpha particle is a result of quantum tunneling. Imagine an alpha particle moving around inside a nucleus, such as thorium (mass number= 232). When the alpha particles bounces against the surface of the nucleus, it meets a barrier caused by the attractive nuclear force. The dimensions of barrier vary a lot from one nucleus to another, but as representative numbers you can assume that the barrier’s width is \( L \approx 35 \) fm (1 fm = \( 10^{-15} \) m) and the average barrier height is such that \( V_0 - E \approx 5 \) MeV. Find the probability that an alpha hitting the nucleus surface will escape. Given that the alpha hits the nuclear surface about \( 5 \times 10^{21} \) times per second, what is the probability that it will escape in a day? The tunneling probability is \( T = e^{-2\alpha L} \) where \( \alpha = \sqrt{2m(V_0 - E)/h} \) and \( L \) is the barrier length. (1 MeV= \( 10^6 \) eV). [5 Marks]