Magnetic fields, the Hall effect and electromagnetic induction
(Electricity and Magnetism)

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Why does a magnet rotate a current carrying loop placed close to it? Why does the secondary winding of a transformer carry a current even when it is not connected to a voltage source? How does a bicycle dynamo work? How does the Mangla Power House generate electricity? Let's perform a simple experiment to investigate some phenomena which can point us towards answers.

KEYWORDS

Faraday's Law · Magnetic Field · Magnetic Flux · Induced EMF · Magnetic Dipole Moment · Hall Sensor · Solenoid

1 Conceptual Objectives

In this experiment, we will,

1. understand one of the fundamental laws of electromagnetism,
2. understand the meaning of magnetic fields, flux, solenoids, magnets and electromagnetic induction,
3. learn how to measure magnetic flux and e.m.f.,
4. interpret the physical meaning of differentiation and integration,
5. verify Faraday's law of electromagnetic induction.

2 Experimental Objectives

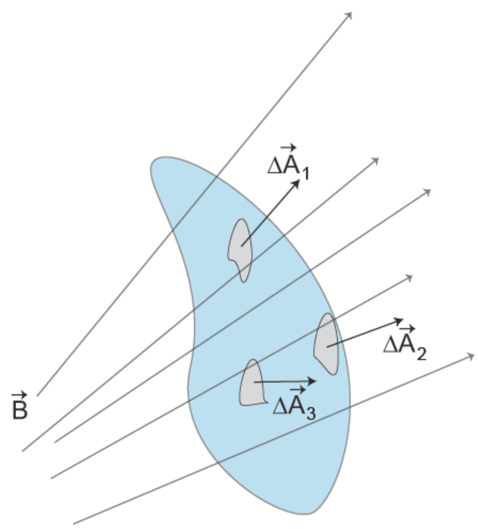
The experimental objective is to use a Hall sensor and a solenoid, to find the magnetic field, magnetization and magnetic flux from static and moving magnets. We will also gain practical knowledge of,

1. magnetic field transducers,
2. visually and analytically determining the relationship between induced emf and magnetic flux,
3. using the sound card as a data acquisition device, and
4. dependence of emf on speed of flux change, hence a direct verification of Faraday's law.

3 The Magnetic Field B and Flux Φ

Around 1821, Oersted found that a current carrying conductor produces a magnetic field. In the twentieth century, scientists determined the configuration of elementary particles in atoms and they realized that electrons inside atoms also produce tiny magnetic fields. This intrinsic magnetic field is found in all materials. The magnetic field is mapped out by the concept of *magnetic field lines*.

Magnetic field lines are like stretched rubber bands, closely packed near the poles .This is why the closer we get to the poles of a magnet, the higher the magnetic field. The number of magnetic field lines passing through an area is known as the *magnetic flux* ϕ.



Consider the arbitrarily shaped surface drawn above immersed inside a magnetic field $\vec{B}$. We want to find the flux through identical area elements $\Delta\vec{A}$; perpendicular and away from the surface. A scalar product between the magnetic field vector $\vec{B}$ and each $\Delta\vec{A}$ is formed and composed to give,

$$\phi = \vec{B}_1 \cdot \Delta\vec{A}_1 + \vec{B}_2 \cdot \Delta\vec{A}_2 \ldots, \qquad (1)$$

which can also be written as ,

$$\phi = \sum_i \vec{B}_i \cdot \Delta\vec{A}_i \tag{2}$$

In the limit of infinitesimal area elements, the flux is

$$\phi = \int \vec{B} \cdot \Delta\vec{A}\, dx \tag{3}$$

4 Electromagnetic Induction

Extensive work was done on current carrying conductors in the nineteenth century, the major ground work was set by Faraday (1831) and following him, Lenz (1834) [1]. Faraday discovered that a changing magnetic field across a conductor generates an electric field. When a charge moves around a closed circuit this electric field does work on the charge. Like the electromotive force (EMF) of a battery this *induced EMF* is capable of driving a current around the circuit.

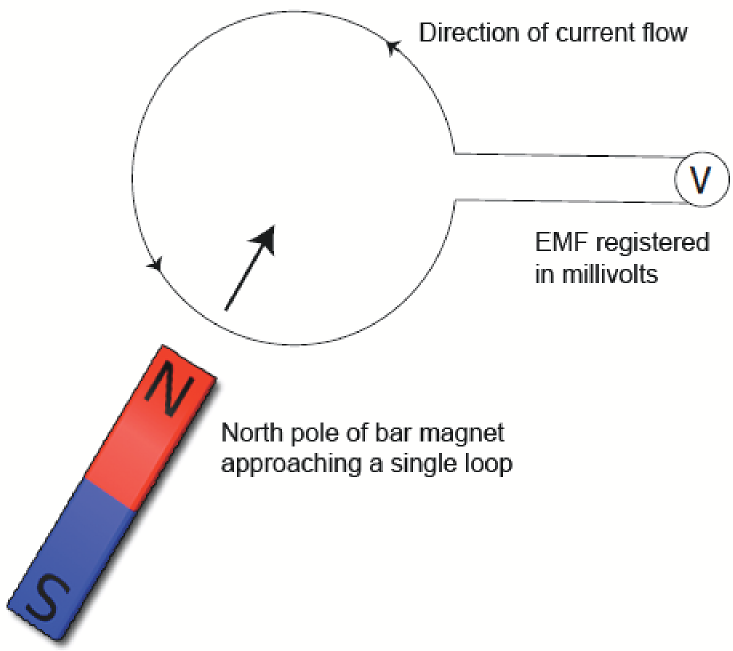


Figure 1: Principle of electromagnetic induction. As a dipole (such as a magnet) is pulled towards a loop of wire, the flux through the loop changes inducing an EMF and current through the loop.

Faraday's law asserts that the EMF produced is directly proportional to the rate at which the magnetic field lines per unit area or magnetic flux 'cuts' the conducting loop. Lenz's law is incorporated into Faraday's Law through a negative sign indicating that the EMF produced opposes the relative motion between the conductor and magnet, it tries to resist the change in flux.

Mathematically both of these laws are expressed together as,

$$\varepsilon = -\frac{d\phi}{dt}, \tag{4}$$

for a single loop of conductor, where ε is the electromotive force induced, ϕ is the magnetic flux. $d\phi/dt$ is time rate of change of magnetic flux. The rate depends on the speed at which the magnet moves relative to the conductor loop, as well as the strength of the field.

Electric power plants or more commonly; generators, are a physical manifestation of the laws of induction. The principle is to change the magnetic flux over large stationary coils. The 'change' of flux is brought about mechanically, for example by falling water or by running a turbine. The changing flux induces an EMF in the coils.

Q 1. What are the units of ε and ϕ?

Q 2. Rewrite Equation 4 for N number of loops. How does the EMF depend on N?

5 Solenoids

It is a coil of wire wound around a core. Magnetically it behaves like a bar magnet, producing a magnetic field when it carries current. It remains a magnet till the time current is flowing through the conductor.

The mathematical expression for magnetic field generated inside an *ideal solenoid* is,

$$B = \mu_0 n I, \tag{5}$$

where μ_0 is the permeability in free space, value; $4\pi \times 10^{-7}\ Tm/A$, n is the number of turns of the conductor per unit length and I is the current through the conductor. The magnetic field B is measured in Tesla (T) or Gauss (G), where 1 G equals 10^{-4} T.In our experiment we will use a changing magnetic field near a solenoid to induce an EMF in it. This is the the Faraday effect!

6 The Hall effect

Imagine a sea. There is a sea of electrons in a conductor. When we apply a potential this 'sea' flows from the higher to the lower potential. Further, if we place this conductor, in which current is flowing in a magnetic field the moving charges tend to interact with the applied magnetic field and also deflect. This deflection results in a potential difference across or perpendicular to the conduction path, know as the *Hall voltage*.

Figure 2 illustrates how moving charges are deflected due to the applied magnetic field. The magnitude of this force (F_B) is given by,

$$F_B = Bq\upsilon, \tag{6}$$

where q is the charge and υ is the velocity. The build-up of charges on one side generates an electric field ($E_\perp$) perpendicular to the current as shown in Figure 2. These charges continue to accumulate till the time force (F_E) due to electric field,

$$F_E = qE_\perp, \tag{7}$$

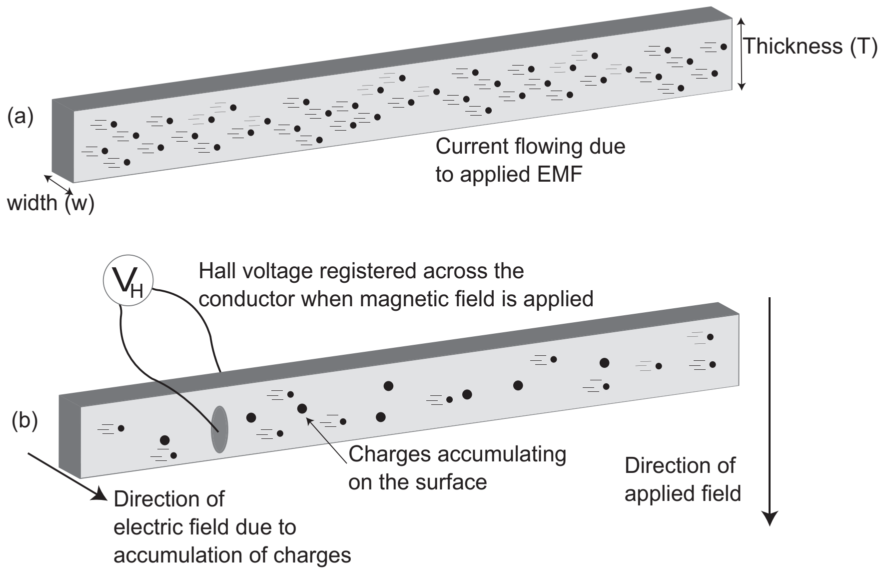


Figure 2: (a) Shows electrons flowing through the conductor. (b) Shows some charges accumulating on the front and back surfaces generating Hall voltage across the width w.

is equal to the force due to the magnetic field (F_B). Mathematically this equilibrium means that,

$$F_E = F_B, \tag{8}$$

or

$$Bqv = qE_{\perp}. \tag{9}$$

The voltage developed due to $E_{\perp}$ is,

$$V_H = E_{\perp}w, \tag{10}$$

where (V_H) is the *Hall voltage* and w is the width of the conductor. Combining Equation 9 and Equation 10 we get,

$$V_H = vwB. \tag{11}$$

We know that the average velocity of electrons in terms of current (I) is given by,

$$v = \frac{I}{neA}, \tag{12}$$

where n is the volume density of electrons and A is the cross-sectional area, a product of width (w) and thickness (T).

Combining Equation 12 and Equation 11 we obtain the Hall voltage in terms of applied magnetic field,

$$V_H = \frac{BI}{neT}. \tag{13}$$

The Hall effect is important in the study of materials, for example it helps us to find the number of conducting particles in a wire and their charge. In our experiment, this effect holds a central importance as we will use sensors developed using this principle to probe the magnetic fields generated by magnets. Read heads in tape recorders and magnetic disk drives utilize this principle too.

Q 3. A strip of copper 150 μm thick is placed inside a magnetic field $B = 0.65$ T perpendicular to the plane of the strip, and a current $I = 23$ A is setup in the strip. What Hall potential difference would appear across the width of the strip if there were 8.49×10^{28} electrons/m^3?

6.1 Interrelationship between the operation of the solenoid and the Hall probe

The Hall probe and the solenoid are both transducers, they convert one form of energy to another. Figure 3 shows that Hall probes generate a measurable potential which varies with the direction and magnitude of the field and flux. This electric potential is then converted to magnetic field using a simple relation provided by the manufacturer of the Hall sensor.

On the other hand, a solenoid, directly measures the EMF which depends on the rate of change of flux and also depends on the number of turns of the solenoid.

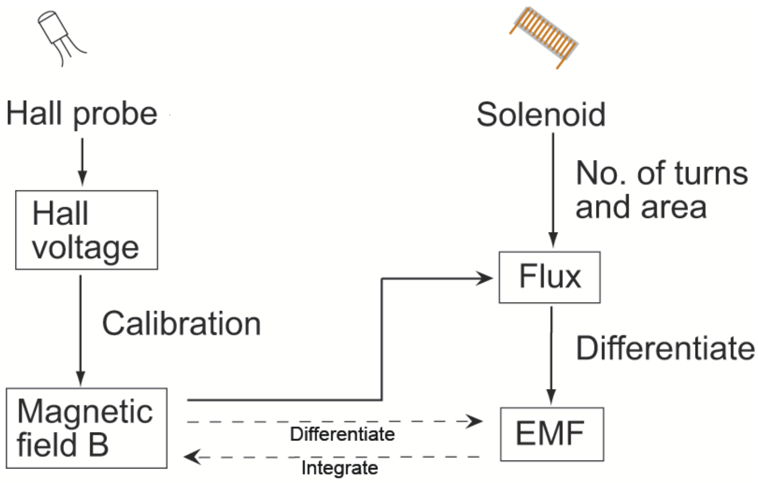


Figure 3: An inter-relationship between the operation of the solenoid and Hall probe.

7 The Experiment

7.1 Understanding the Hall sensor

Hall sensors are used to measure magnetic fields. In our case, we've provided you with a sensor mounted on the tip of a probe. The output voltage of a Hall sensor V_H is proportional to the magnetic field B being measured. The measured voltage is then converted to magnetic field using a calibration scheme provided by the manufacturer of the Hall sensing chip. This calibration curve is shown in Figure 5. The relation between the hall voltage V_H and the magnetic field is

$$B = 320V_H - 800 \tag{14}$$

where V_H is in volts and B is in Gauss

The voltage V_{cc} is provided to the Hall sensor using the Universal Serial Bus (USB) port. All USB ports have a 5 V regulated output; we will be using USB port as power supply to the Hall chip.

7.2 Magnetic field of a disk magnet

In the first step, we will map the field of a disk magnet using the Hall probe. You are provided with small disk magnets.

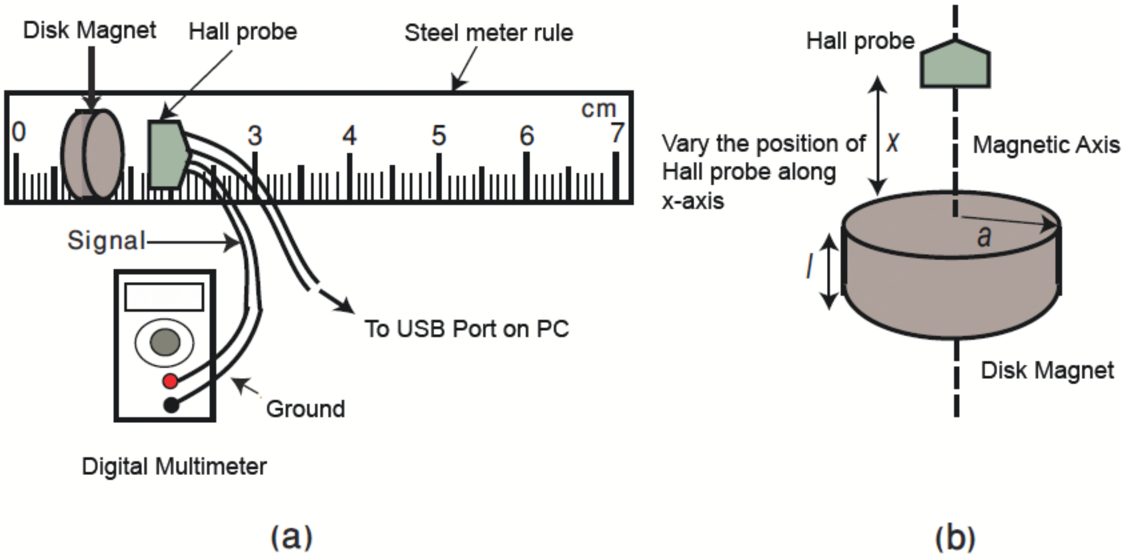


Figure 4: Schematic diagrams. (a) Hall probe voltage measurement, (b) disk magnet field mapping using a Hall probe.

Q 4. Keeping the hall probe away from the magnet, connect the USB port to the computer, also connect the **signal** and **ground** wires to the provided digital multimeter. Note that the digital multimeter is displaying 2.5V as you connect the Hall probe. Using Equation 14, determine what magnetic field does this voltage correspond to.

Q 5. Attach a disk magnet to the provided steel meter rule and vary the distance between the disk magnet and the flat face of the Hall probe. Make sure that the flat face of the probe is perpendicular to the magnetic axis of the disk magnet as illustrated in Figure (4b). Vary the distance from 0 to 35 mm in steps of, say, 5 mm.

Q 6. Record the output voltage on the digital multimeter as you move the Hall probe along the magnetic axis, and make a table as suggested in Table 1 given below.

Q 7. Find the corresponding magnetic field strength using the calibration curve given in Figure (5).

Q 8. Find the uncertainty in measured magnetic field $B_{measured}$.

Q 9. Plot a graph between magnetic field strength $B_{measured}$ and distance x. Interpret and describe the plot.

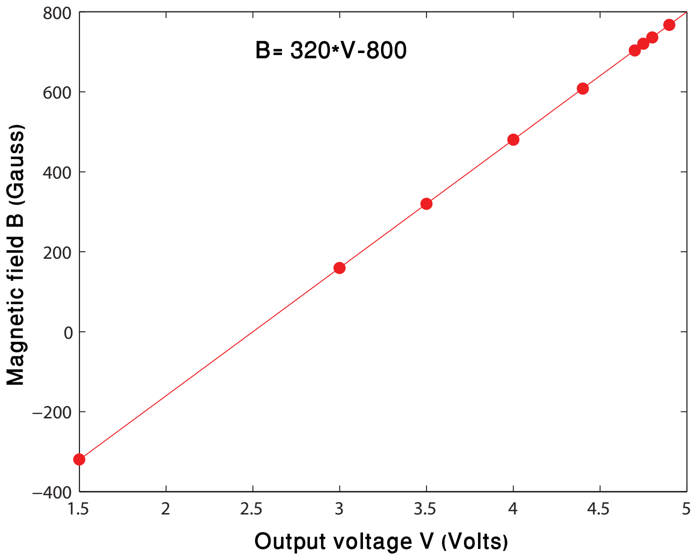


Figure 5: Calibration curve of the Hall sensor employed in the experiment

Distance (mm) x	Output Voltage (V)	Measured magnetic field $B_{measured}$ (G)	$B_{measured}$ (T)	f(x) (T m/A)
0				
2				
.				
.				

Table 1: Mapping the field of a disk magnet. For voltage to field conversion use calibration sheet. For $f(x)$, see section 7.3. Note $G = 10^-4T$.

7.3 Magnetization of a disk magnet

In this part of the experiment, we will use the obtained data to estimate the magnet's magnetization. Magnetic materials are made up of atoms which have *magnetic dipole moment* $\vec{\mu}_B$. These randomly aligned dipoles have a resultant magnetic dipole $\vec{\mu}$,

$$\vec{\mu} = \sum \vec{\mu}_B. \tag{15}$$

We define magnetization $\vec{M}$ as

$$\vec{M} = \frac{\sum \vec{\mu}_B}{V} \tag{16}$$

where V is the volume of the material.

For a disk magnet the expression for the magnetic field strength as a function of distance is approximately,

$$B(x) \approx \frac{\mu_0 M}{2}\left(\frac{x + l/2}{\sqrt{(x + l/2)^2 + a^2}} - \frac{x - l/2}{\sqrt{(x - l/2)^2 + a^2}}\right), \tag{17}$$

where M is the magnetization of the disk magnet, x is the distance along its magnetic axis, l is its thickness and a is its radius [4].

The term in brackets needs some mathematical detail in which we will not delve. However it is important to tell that it is a geometrical term which is the result of an integral depending upon the dimensions of the magnet and solved over the distance at which we are measuring the field.

For the sake of simplicity lets replace

$$\frac{\mu_0}{2}\left(\frac{x+l/2}{\sqrt{(x+l/2)^2+a^2}}-\frac{x-l/2}{\sqrt{(x-l/2)^2+a^2}}\right) \tag{18}$$

with the geometrical function $f(x)$, obtaining

$$B(x) = Mf(x). \tag{19}$$

The goal is to find the magnetization of the disk magnet using Equation 19.

Q 10. For this purpose perform the following procedure.

1. Find the thickness of the disk magnet with vernier callipers.
2. Find its radius.
3. Run Matlab.
4. Construct a vector showing the values of x at which the magnetic field is measured. Suppose the vector is labeled x. Type >> **magneticfield**(**x**);
5. The program prompts to enter the radius and thickness in mm.
6. The Matlab code returns the value of $f(x)$ given by the expression 17. Tabulate these values in your notebook. See last column of 1.

Q 11. Make a plot between $f(x)$ and $B_{measured}$ and find the magnetization. Use weighted fitting technique. You could assume zero uncertainty in x and $f(x)$. The typical magnetization of iron is in the range $\approx 10^6$ A/m [6].

7.4 Verification of Faraday's law of electromagnetic induction

In the second part, we will compare signals derived from a hall sensor and a solenoid as disk magnets negotiate and from beneath move in their vicinity. The experimental setup consists of a rotating disk. Magnets are placed at fixed intervals.

In one case, we will use a Hall sensor and in the other we will use the solenoid. We will then compare the signals and observer the dependance on the speed of rotation, stacking magnets on top of the another etc.

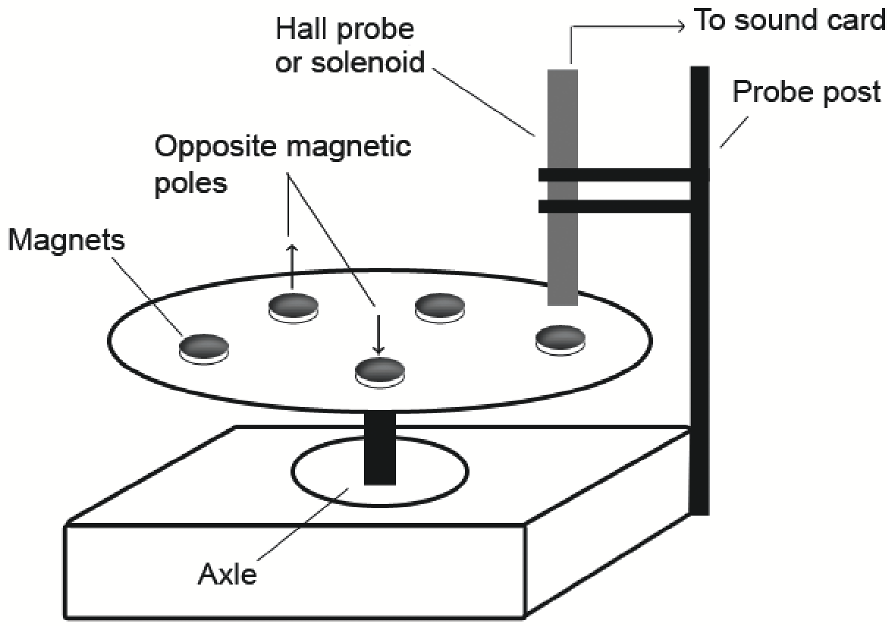


Figure 6: Setup of the experiment. The magnets can be perched in two configurations, with their magnetizations pointing up or down.

7.5 Data acquisition using a sound card

In this experiment, data acquisition (DAQ) is done using a sound card. The sound card can be used both for data acquisition and signal generation. The sound card is, in fact, a simple analog to digital converter, usually compatible with the input and output of audio signals in the range of 20 Hz to 20 KHz. Thus, the signals upto 20 KHz can be easily acquired and generated using the sound card.

The sound card has a 3.5 mm TRS stereo jack connector for generating and acquiring signals as depicted in Figure (7). The right and left channels of the TRS sereo connector can be used as analog input (AI), analog output (AO), or one as AI and other as AO. We will use the left channel of the sound card for acquiring the signal.

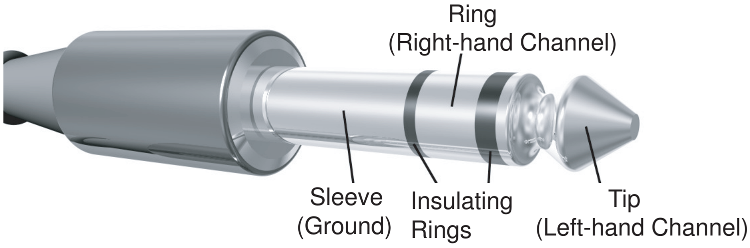


Figure 7: TRS stereo jack connector.

7.6 Observing the induced EMF flux using the solenoid

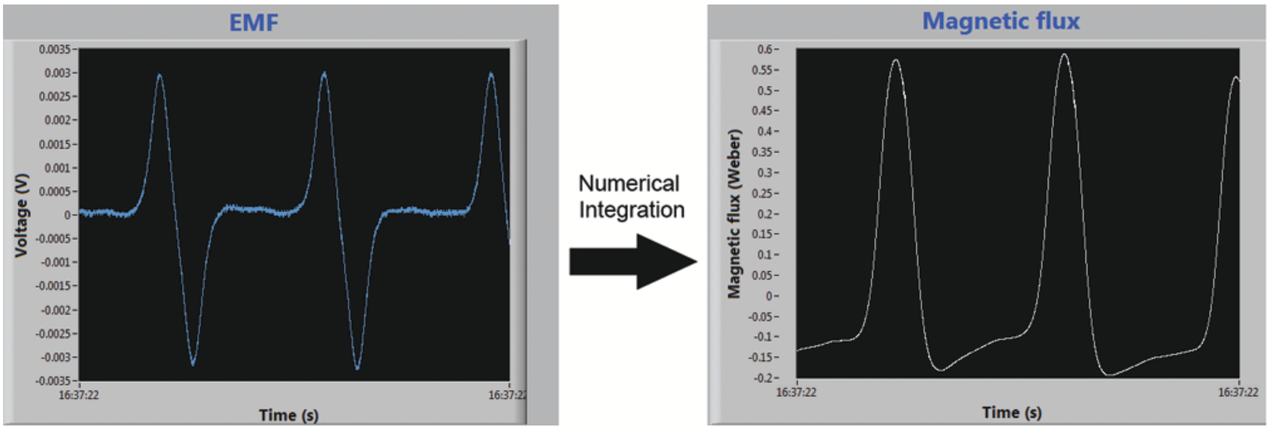


Figure 8: (a) EMF induced in the solenoid as the magnet passes below the solenoid and (b) Shows the numerical integration of the EMF.

Q 12. Carry out the following procedure.

1. Note the number of turns of the provided solenoid.
2. Align the solenoid with the magnetic field of the magnets.
3. You have been provided with a box for the sound card circuitry with labeled **Solenoid** and **Sound Card** ports. Connect the hardware to the appropriate ports.
4. Rotate the disk manually and run the labview file named **“solenoid.vi”**.
5. Click on the **Run** button, the data starts acquiring.
6. Now, observe the waveform graphs.
7. Observe the EMF on the graph labeled as EMF.
8. Observe the magnetic flux on the graph labeled as Magnetic flux.

Q 13. You will observe something similar to Figure 8. Is there a mathematical relation between the graphs you see? Figure 8(b) is just a numerical integration of the emf developed.

7.7 Observing magnetic field using the Hall sensor

Now instead of the solenoid we'll use the Hall probe as a magnetic field transducer. The voltage across the Hall chip is proportional to the magnetic flux. Depending on the front (flat) or the back (round) surface or the direction of the field, voltage will either drop below or jump above the voltage when there is no field present.

Q 14. Now carry out the following procedure to observe the behavior of the Hall Probe.

1. Place the Hall probe sensor about 2 cm above the plane if the magnets.
2. You have been provided with the potential divider circuit on a vero board, enclosed inside a box with labeled **Hall Probe** and **Sound card** ports. Connect the Hall probe **signal** and **ground** leads to the labeled terminals of the sound card, also connect the sound card port to the computer.
3. Rotate the disk manually and run the **"hallprobe.vi"** file.
4. Click on the **Run** button, the data starts acquiring.
5. Now, observe the waveform graphs.
6. Observe the Hall voltage on the graph labeled as voltage.

Q 15. Note down all your observations and inferences in your note books.

Q 16. What's the difference you observe when using the Hall probe sensor from the solenoid?

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