Tutorial on Position and Momentum

1. For a particle whose wave function is given by

\[ \psi(x) \propto \frac{1}{a^2 + x^2}. \]

(a) Normalize the wave function.
(b) Calculate \( P(-a \leq x \leq a) \).
(c) Calculate \( \langle x \rangle \) and \( \Delta x \).
(d) Find the momentum representation of the state of the particle. Use this representation to calculate \( \langle p \rangle \) and \( \Delta p \) and verify the indeterminacy relation.

2. Show that

\[ F \left\{ \frac{1}{\sqrt{a}} \text{rect} \left( \frac{x}{a} \right) \right\} = \sqrt{\frac{a}{2\pi \hbar}} \text{sinc} \left( \frac{pa}{2\hbar} \right), \]

where the rectangle function is defined as

\[ \text{rect}(x) = \begin{cases} 
1, & |x| < 1/2, \\
0 & \text{elsewhere},
\end{cases} \]

and

\[ \text{sinc}(p) \equiv \frac{\sin(p)}{p}. \]

Calculate \( \langle x \rangle, \Delta x, \langle p \rangle, \Delta p \) and plot the wavefunctions.