Measurement of thermal expansion coefficients using a strain gauge

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A strain gauge is employed to determine the coefficient of thermal expansion of a solid. The apparatus is interfaced to a computer to obtain a linear plot of displacement against temperature.

I. INTRODUCTION

A classic experiment for the introductory laboratory has been the measurement of the thermal expansion coefficient of a metal.[1,2] The apparatus normally consists of a steam jacket housing a metal rod. The rod, heated by steam, passes through the end of the jacket and completes an electrical circuit after expanding a predetermined amount. The coefficient of thermal expansion is then taken to be the fractional change in the length of the rod per change in temperature. Over the years, this very fundamental measurement of solid-state physics has often fallen from the general physics laboratory repertoire in favor of experiments considered to be more modern.

Because of the current interest in new materials and their physical properties, we sought to update the technique for the measurement of thermal expansion in an introductory laboratory, employing a technique that makes use of continuous data recording by a computer. Research methods for the measurement of dimensional changes in solids normally employ sensitive capacitance dilatometers,[3] although some researchers have used strain gauges.[4] The dilatometer requires expensive and elaborate apparatus, whereas a strain gauge, if used in a rather unconventional fashion, serves our purpose nicely. Used in connection with a Wheatstone bridge interfaced to a personal computer, the strain gauge serves as a useful pedagogical device for the introductory laboratory and brings back to the introductory laboratory an experiment that has been absent for too long. It also introduces the student to the strain gauge, a device widely used in industry for the measurement of pressure, biomechanical stress, etc.[5]

II. THE APPARATUS

A strain gauge[6] of 120-Ω nominal resistance is attached to a sample of either steel, copper, aluminum, or Plexiglas using the cold-curing Z-70 adhesive supplied by the manufacturer. Three wires of Advance, a copper–nickel resistance alloy, were connected to the strain gauge (see Fig. 1), and the sample with its attached gauge was placed in an oven or heated oil bath. The Wheatstone bridge circuitry used for the experiment is shown in Fig. 1. The variable resistor arm of the bridge consists of a 47-Ω resistor in series with a ten-turn 100-Ω potentiometer used to adjust for zero voltage between points A and B of the bridge.

The arrangement of the three resistance alloy lead wires attached to the strain gauge is important. The arrangement minimizes the voltage variation due to the temperature dependence of the lead wire resistance. The lead wires were chosen to be a resistance alloy since it has a very small temperature coefficient of resistance. The placement of these lead wires as shown in Fig. 1 ensures that any changes in the lead wire resistance with temperature will be shared equally by the two branches and will not alter the bridge voltage V_{out}.

The output voltage from the bridge was amplified with a precision instrumentation amplifier, an AD524A. This amplifier, based on the classic three op-amp circuit, is commonly used in bridge and thermocouple applications that require a high differential input resistance (10^9 Ω for the AD524A).[7] The amplification is pin selectable and has a maximum gain error of ± 2%.

The samples of steel, copper, aluminum, and Plexiglas were commercial pieces whose properties are not known precisely.

The sample temperature was measured using a thermistor network that has a nominal resistance of 2764 Ω at 0 °C.[8] The thermistor network produces a linear voltage output over the temperature range from 0 to 100 °C. Deviation from linearity of the thermistor network does not exceed ± 0.05 °C. The thermistor bead was attached to the sample by means of varnish.

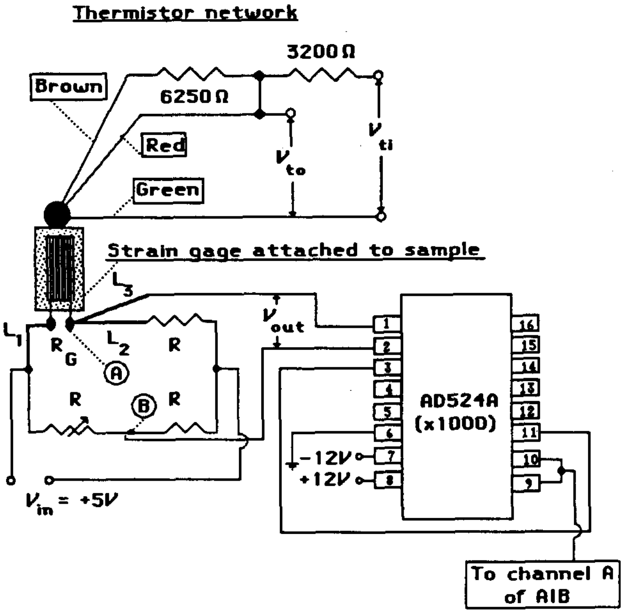

Fig. 1. Circuit diagrams for the strain gauge and linear thermistor network. The thermistor network voltage V_{to} is monitored through channel B of the AIB. The AD524A is an instrument amplifier with a precision gain of 1, 10, 100, or 1000. The leads L_1, L_2, and L_3 are made of a resistance alloy.

III. THEORY

A strain gauge is constructed by evaporating a thin Constantan foil unto a thin insulating backing of Polyimid, a plastic material. If the gauge undergoes an expansion due to a change in temperature, the resistance of the gauge will be altered. The gauge is constructed so as to respond primarily to the expansion along a given dimension, and the gauge is matched to the thermal expansion of particular materials. For example, the gauge used in our experiment was matched to the thermal expansion of ferritic steel whose coefficient of thermal expansion is $11\times10^{-6}/^\circ\mathrm{C}$.

If we assume that the resistance of each arm of the Wheatstone bridge shown in Fig. 1 is the same, the voltage output between points A and B of the bridge is given as

$$V_{\mathrm{out}} = [V_{\mathrm{in}}/(R+R_G)]R_G - (V_{\mathrm{in}}/2R)R,$$

where V_{in} is the voltage applied to the bridge and R_G is the strain gauge resistance. When R_G is equal to R, the output voltage from the bridge is zero. If the resistance of the strain gauge changes from R to $R+\delta$ due to the expansion of the sample with a change in temperature, the equation for the voltage output from the bridge becomes

$$V_{\mathrm{out}} = V_{\mathrm{in}}[(R+\delta)/(2R+\delta)-\tfrac{1}{2}].$$

Assuming that $\delta \ll R$, the equation reduces to

$$V_{\mathrm{out}} \approx V_{\mathrm{in}}[\delta/(4R)]. \qquad (1)$$

Again, this equation assumes that all the resistors R are initially equal to each other. In our setup we used 1% precision resistors.

IV. DATA AND ANALYSIS

Data were recorded for samples of Plexiglas, aluminum, copper, and steel. An advanced interface board (AIB) was used to acquire the data by computer.[9] The thermistor voltage corresponding to the temperature and the output voltage of the bridge were read every 10 s through two analog inputs to the interface board as the sample cooled from 100 °C to room temperature. Data were taken on the cooling cycle to ensure better temperature uniformity throughout the sample. A typical plot of the data for aluminum is shown in Fig. 2.

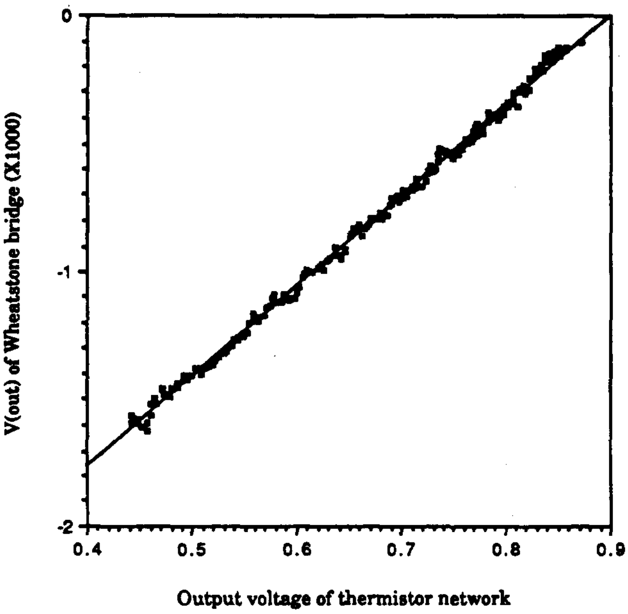

Fig. 2. Wheatstone bridge output voltage V_{out} plotted against thermistor output voltage V_{to}. The strain gauge was attached to a sample of commercial aluminum in this case and the AD524A was set to a gain of 1000. The value of the slope, $\Delta V_{\mathrm{out}}/\Delta V_{\mathrm{to}}$, is (3.55/1000) for this case.

It is obvious from the data plotted in Fig. 2 that the length of the sample varies in a linear fashion with temperature, that is, that the thermal expansion coefficient of the material is approximately constant in this temperature range. If all that is desired is a classroom demonstration of this fact, a real-time plot of the data over a period of a few minutes would suffice. However, very reasonable results for actual thermal expansions can be obtained by the following calculation.

The fractional change in the resistance of a strain gauge δ/R_G is related to the fractional change in the length of the gauge $\Delta L/L$ through the expression

$$\delta/R_G = [\mathrm{GF}]\times(\Delta L/L), \qquad (2)$$

where GF is the strain gauge factor. For our particular gauge, the gauge factor supplied by the manufacturer is $\mathrm{GF} = 2.02 \pm 1\%$.

The thermistor calibration equation, supplied by the manufacturer, is

$$\Delta V_{\mathrm{to}} = (0.005\,348\,3V_{\mathrm{ti}})\Delta T,$$

where ΔV_{to} is the output voltage of the thermistor due to a temperature change ΔT, and V_{ti} is the voltage applied to the thermistor network. Combining this expression with that of Eqs. (1) and (2) yields

$$\alpha = \left(\frac{\Delta L/L}{\Delta T}\right) = \left[\frac{1}{\mathrm{GF}}\left(\frac{4\Delta V_{\mathrm{out}}}{V_{\mathrm{in}}}\right)\bigg/\frac{\Delta V_{\mathrm{to}}}{0.005\,348\,3V_{\mathrm{ti}}}\right] = \mathrm{const}\times\left(\frac{\Delta V_{\mathrm{out}}}{\Delta V_{\mathrm{to}}}\right), \qquad (3)$$

where $\Delta V_{\mathrm{out}}/\Delta V_{\mathrm{to}}$ is the slope of the data plotted in Fig. 2. Note that the voltage V_{out} used in this expression is the voltage *before* amplification by the instrumentation amplifier.

It must be remembered that the strain gauge is constructed to be matched to the thermal expansion of ferritic steel which has a thermal expansion coefficient of $11\times10^{-6}/^\circ\mathrm{C}$. One could, to a rough approximation, assume that the small value for the slope (0.0006) for steel is approximately zero. In that case the final expression for the coefficient of linear expansion is

$$\alpha \sim \mathrm{const}\times\left(\frac{\Delta V_{\mathrm{out}}}{\Delta V_{\mathrm{to}}}\right) + \alpha_{\text{ferritic steel}}. \qquad (4)$$

This is made clear in Fig. 3 where we have plotted the coefficients of thermal expansion for Plexiglas, aluminum, copper, and ferritic steel against the slope $(\Delta V_{\mathrm{out}}/\Delta V_{\mathrm{to}})$ obtained for each sample. A simple linear fit to the data in Fig. 2 yields a value of $22.4\times10^{-6}/^\circ\mathrm{C}$ for the coefficient of thermal expansion of aluminum using Eq. (4). The value for the thermal expansion of pure aluminum is $23\times10^{-6}/^\circ\mathrm{C}$.

Overall, this experiment has a number of interesting facets that are thought provoking and instructive for the student. The apparatus for this experiment, except for the versatile interface, is rather inexpensive and simple to set up.

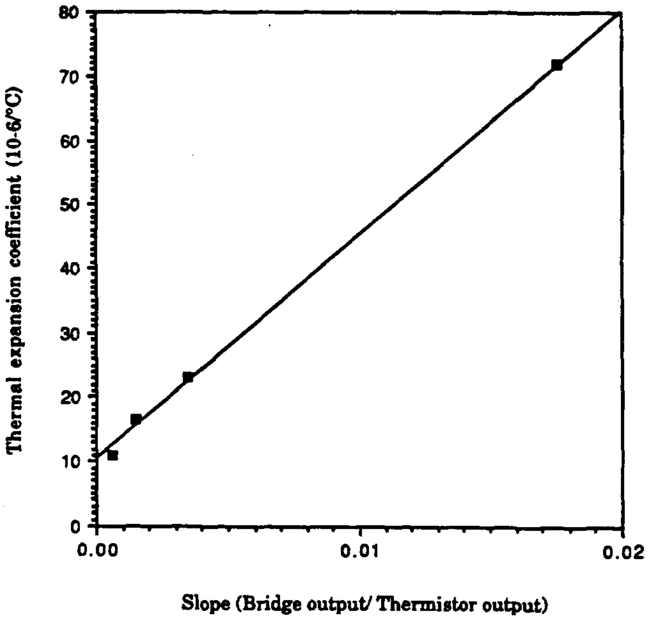

Fig. 3. Relation between the known values for the coefficients of thermal expansion and the slope of the data from graphs such as that of Fig. 2. The coefficients are $\alpha_{\mathrm{Fe}} = 11\times10^{-6}$/°C, $\alpha_{\mathrm{Cu}} = 16.6\times10^{-6}$/°C. $\alpha_{\mathrm{Al}} = 23\times10^{-6}$/°C, and $\alpha_{\mathrm{Plexiglas}} = 72\times10^{-6}$/°C.

In a matter of minutes, the apparatus gives a clear visual display of the relation between the thermal expansion of a solid and the temperature in the temperature range from 40 °C to 100 °C. Data for a series of materials can be obtained easily during a laboratory period. If presented as an independent study project, students will have the time necessary to perform their own calibration of the thermistor network. Because of the flexibility of the apparatus, the ease of data accumulation, and the simplicity of the concepts involved, the apparatus offers a unique opportunity for independent research by students at the introductory level. Finally, the experiment presents an informative use of the Wheatstone bridge in a practical application.

ACKNOWLEDGMENTS

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[1] L. R. Ingersoll, M. J. Martin, and T. A. Rouse, *A Laboratory Manual of Experiments in Physics* (McGraw-Hill, New York, 1953), 6th ed., pp. 88–90.

[2] *The Taylor Manual—Advanced Undergraduate Laboratory Experiments in Physics*, edited by T. B. Brown (Addison-Wesley, Reading, MA, 1961), pp. 83–90.

[3] K. Tanimura, N. Itoh, K. Arima, H. Imaike, and D. W. Heikkinen, "Capacitance dilatometer for *in situ* measurement of radiation-induced dimensional changes in solids," Rev. Sci. Instrum. **58**, 1284–1286 (1987).

[4] A. M. Kinan, "Utilization of foil resistance strain gauges to measure the thermal expansion anisotropy of several types of beryllium," in *Thermal Expansion—1973*, American Institute of Physics Conference Proceedings No. 17, edited by R. E. Taylor and G. L. Denman (American Institute of Physics, New York, 1974).

[5] See, for example, "Practical strain gauge measurements," Omega Engineering, Inc., One Omega Drive, Box 4047, Stamford, CT 06907.

[6] The strain gauge used is a model LY41-6/120 (6-V maximum, 120 Ω) matched to steel. It is available from Omega Engineering, Inc.

[7] The precision instrument amplifier AD524A is available from Analog Devices, One Technology Way, Norwood, MA 02062-9106.

[8] Model YSI 44201, available from Yellow Springs Instrument Co., Yellow Springs, OH 45387.

[9] Advanced Interface Board manufactured by Sunset Laboratories and available through Vernier Software, 2920 S.W. 89th Street, Portland, OR 97225.

Is the time gone forever when, aroused by his inner freedom and the independence of his thinking and his work, [the man of science] has a chance of enlightening and enriching the lives of his fellow human beings? In placing his work too much on an intellectual basis, has he not forgotten about his responsibility and dignity?

Albert Einstein, *Ideas and Opinions* (Crown, New York, 1954).