Observation of Elliptically Polarized light using Quarter Waveplate

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1 Introduction

In this experiment we will study the change in the polarization of light from being linearly polarized to circularly/elliptically polarized light while passing through birefringent materials. The experimental setup consists of polarizer, quarter waveplate, photodiode and oscilloscope. It will be analyzed using Jones calculus to yield the expression for the intensity of the transmitted beam.

1.1 Polarization of Light

In wave picture of light it is considered to consist of oscillating electric and magnetic field. The oscillations of electric and magnetic field as well as the direction of propagation are mutually orthogonal and any of the two fields is sufficient for complete description of light. As is the convention we will describe it using electric field. The electric field can have different orientations in space which determine its polarization. Let us restrict ourselves to the case of a harmonic wave traveling along the z-axis. The electric field of such a wave will lie in the x-y plane and we can represent it as

$$\overrightarrow{E} = \hat{x}E_{ox}e^{i(kz-\omega t)} + \hat{y}E_{oy}e^{i(kz-\omega t+\Gamma)} \tag{1}$$

Depending upon the value of Γ we can divide the polarization in two types

1.1.1 Linear Polarization

If the value of Γ in (1) is 0 or a multiple of $\pm\pi$ then polarization will be linear as the direction of electric field remains fixed. The direction of polarization is described by the angle $\theta = \tan^{-1}(\frac{E_{oy}}{E_{ox}})$.

1.1.2 Elliptical Polarization

If the value of Γ is anything but 0 or multiple of $\pm\pi$ then the direction of polarization is not fixed as the electric field will sweep out an ellipse during each period. Due to this it is called elliptical polarization. If Γ is positive then the electric field will rotate in the counter-clockwise direction and the wave is said to be left elliptically polarized. If Γ is negative the electric field will rotate in the clockwise direction and the wave is said to be right elliptically polarized.

1.1.3 Circular Polarization

Circular polarization is the special case of elliptical polarization which is obtained when $E_{ox} = E_{oy}$ and $\Gamma = \pm\frac{\pi}{2}$.In this case the electric field will sweep out a circle in each period. The wave is said to be left or right circularly polarized if the direction of rotation of electric field is counter-clockwise or clockwise respectively. The phase-difference Γ between x and y

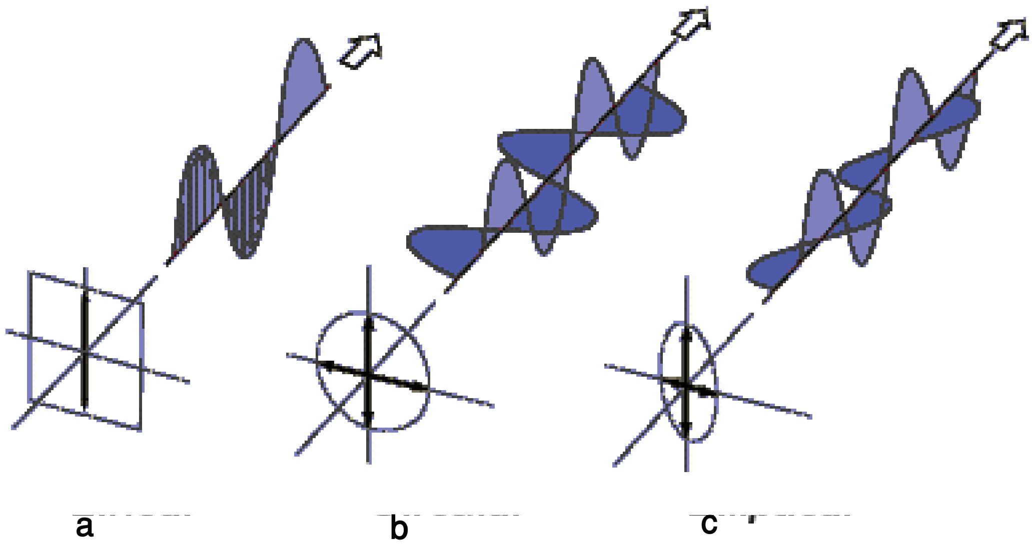


Figure 1: Polarizations of Light a)Linear b)Circular c)Elliptical

components of electric field can be obtained by using birefringent materials which we will now discuss.

1.2 Birefringence

This experiment uses wave plates which use the property of birefringence. Birefringent materials have anisotropic index of refraction which means that it depends on both propagation and polarization direction. This effect is due to asymmetries in the structure of these crystals. So highly symmetric cubic crystals like NaCl do not show this behavior but it is shown by hexagonal crystals like calcite and quartz etc.[1] An unpolarized ray that is incident normally on a birefringent material is refracted into two rays. One of the rays obey's Snell's law and is called ordinary ray. The index of refraction characterizing this ray is denoted by n_o. The other ray does not obey Snell's law and is called extraordinary ray and its index of refraction is denoted by n_e[2].

To describe the orientation of such a birefringent crystal we use the concept of optic axis which is defined as the direction in which the transmitted ray does not show birefringence.

Now we consider the simple case of normal incidence on a slab of such material cut so that the optic axis is parallel to the slab face. Such a slab is called a waveplate. In this case a ray polarized parallel to the optic axis propagates with index of refraction n_o and a ray polarized perpendicular to the optic axis propagates with index of refraction n_e. Normally incident light with any linear polarization incident on such a slab can be resolved in to two components, one having polarization perpendicular to the optic axis and other parallel to optic axis propagating with n_e and n_o respectively. As the velocity is a function of index of refraction so the two components travel at different velocities inside the slab. This introduces a phase difference Γ between them which depends on the thickness 'd' of the plate and is given by

$$\begin{aligned}\Gamma &= k_o d - k_e d \\ &= \frac{2\pi n_o d}{\lambda} - \frac{2\pi n_e d}{\lambda} \\ &= \frac{2\pi \Delta n d}{\lambda} \\ \text{where } \Delta n &= n_o - n_e\end{aligned} \tag{2}$$

1.3 Quarter Wave Plate

If the thickness of the slab is adjusted so that this phase difference is an odd multiple of $\frac{\pi}{2}$ then such a slab is called quarter wave plate. So for quarter wave plate

$$\begin{aligned}(2m+1)\frac{\pi}{2} &= \frac{2\pi \Delta n d}{\lambda} \\ \Rightarrow d &= \frac{(2m+1)\lambda}{4\Delta n}\end{aligned}$$

Let a linearly polarized ray given by

$$\overrightarrow{E} = \hat{x}E_{ox}e^{i(kz-\omega t)} + \hat{y}E_{oy}e^{i(kz-\omega t)}$$

be incident normally on a quarterwave plate having optic axis along x-axis, then the transmitted wave would be

$$\overrightarrow{E} = \hat{x}E_{ox}e^{i(kz-\omega t)} + \hat{y}E_{oy}e^{i(kz-\omega t+\Gamma)}$$

for quarter wave plate it would become

$$\overrightarrow{E} = \hat{i}E_{ox}e^{i(kz-\omega t)} + \hat{j}E_{oy}e^{i(kz-\omega t\pm\frac{\pi}{2})} \tag{3}$$

1.3.1 Generating Circularly Polarization using quarter wave plate

If linearly polarized light making an angle of $\frac{\pi}{4}$ with optic axis is incident on a quarter wave plate then E_{ox}=E_{oy}=E_o then the (1) becomes

$$\overrightarrow{E} = \hat{x}E_{o}e^{i(kz-\omega t)} + \hat{j}E_{o}e^{i(kz-\omega t\pm\frac{\pi}{2})}$$

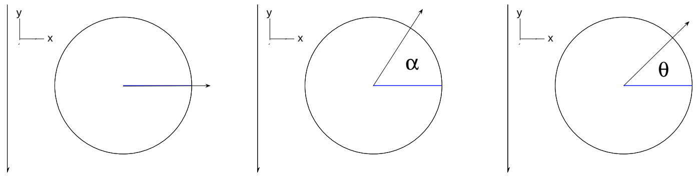
and hence circularly polarized light will be produced.As described earlier if $\Gamma = \frac{\pi}{2}$ then the rotation is counter-clockwise and the wave is said to be left-circularly polarized, else if $\Gamma = -\frac{\pi}{2}$ then the rotation is clockwise and the wave is said to be right-circularly polarized.

1.3.2 Generating Elliptical Polarization using quarter wave plate

If linearly polarized light making angle $\theta \neq \pm\frac{\pi}{4}$ is incident on the quarter wave plate then $E_{ox} \neq E_{oy}$ and hence the transmitted light is elliptically polarized.

2 Analysis Using Jones Calculus

Jones Calculus is a mathematical tool to describe the state of polarization of light. It was invented by R. C. Jones in 1941. In it the polarization of light is described by 2×1 matrices known as Jones vectors. The general form of a Jones vector is $\begin{bmatrix} E_x \\ E_y \end{bmatrix}$, where E_x and E_y are the x and y components of the electric field of the light and it is propagating along z-axis. Different optical components like polarizers, analyzers and waveplates are represented by 2×2 matrices known as Jones matrices. The polarization of transmitted light is obtained



(a) Polarizer (b) Waveplate (c) Analyzer

Figure 2: Relative orientation of axes of Polarizer, Analyzer and Waveplate

by multiplying the Jones matrices of successive components in order with the Jones vector of the incident light. We will use normalized Jones vectors for incident light which have unit magnitude. In this case the vector of transmitted light obtained will give the fraction of incident electric field amplitude transmitted. The amplitude of transmitted ray can be obtained by multiplying with it the amplitude of incident light.[2]

Our system consists of a linear polarizer having transmission axis along x-axis, a wave waveplate whose optic-axis makes an angle α with the x-axis and an analyzer whose transmission axis makes an angle θ with the x-axis.

So the Jones matrix of the system will be

$$H = T_3 T_2 T_1 \tag{4}$$

Where T_1 is the Jones matrix of the Polarizer i.e

$$T_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{5}$$

The Jones matrix of the analyzer can be obtained by considering the case of full and null transmission. Let the beam incident on the analyzer be described by $\begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix}$ and let the Jones matrix of the analyzer be

$$T_3 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{6}$$

When $\theta = \phi$ the transmission is maximum so

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\Rightarrow a\cos\theta + b\sin\theta = \cos\theta \tag{7}$$

$$\text{and } c\cos\theta + d\sin\theta = \sin\theta \tag{8}$$

and when $\phi = \theta + \frac{\pi}{2}$ there is null transmission so

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \cos(\theta + \frac{\pi}{2}) \\ \sin(\theta + \frac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-a\sin\theta + b\cos\theta = 0 \tag{9}$$

$$-c\sin\theta + d\cos\theta = 0 \tag{10}$$

$$\Rightarrow a\sin\theta = b\cos\theta$$

$$\Rightarrow b = a\frac{\sin\theta}{\cos\theta} \qquad \text{from (9)}$$

$$\Rightarrow a\frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \cos\theta \qquad \text{using (7)}$$

$$\Rightarrow a = \cos^2\theta \tag{11}$$

so,

$$
\begin{aligned}
b &= \frac{\cos^2\theta \sin\theta}{\cos\theta} \\
b &= \sin\theta\cos\theta && (12)\\
\text{and } d\cos\theta &= c\sin\theta && \text{using (10)}\\
\Rightarrow d &= c\frac{\sin\theta}{\cos\theta} \\
\text{which gives } c\cos\theta + c\frac{\sin^2\theta}{\cos\theta} &= \sin\theta && \text{using (8)}\\
c\frac{\cos^2\theta+\sin^2\theta}{\cos\theta} &= \sin\theta \\
\Rightarrow c &= \cos\theta\sin\theta && (13)\\
\Rightarrow d &= \cos\theta\sin\theta\frac{\sin\theta}{\cos\theta} \\
d &= \sin^2\theta && (14)\\
\text{so } T_3 &= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix} && \text{from (11),(12),(13) and (14)} \quad (15)
\end{aligned}
$$

The matrix for the wave plate is

$$T_2 = R(\alpha)T_2'R(-\alpha) \tag{16}$$

Where

$$T_2' = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{bmatrix}$$

and

$$R(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

So (16) becomes

$$T_2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \tag{17}$$

Since,

$H = T_3T_2T_1$

From (5),(15) and (17) it becomes

$$
\begin{aligned}
H &= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} \cos^2\theta\cos\alpha + \cos\theta\sin\theta\sin\alpha & -\sin\alpha\cos^2\theta + \cos\theta\cos\alpha\sin\theta \\ \sin\theta\cos\theta\cos\alpha + \sin^2\theta\sin\alpha & -\sin\theta\sin\alpha\cos\theta + \sin^2\theta\cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 \\ -\sin\alpha & 0 \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta(\cos\theta\cos\alpha + \sin\theta\sin\alpha) & \cos\theta(\cos\alpha\sin\theta - \sin\alpha\cos\theta) \\ \sin\theta(\cos\theta\cos\alpha + \sin\alpha\sin\theta) & \sin\theta(\sin\theta\cos\alpha - \cos\theta\sin\alpha) \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 \\ -e^{-i\Gamma}\sin\alpha & 0 \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta\cos(\theta-\alpha) & \cos\theta\cos(\theta-\alpha) \\ \sin\theta\cos(\theta-\alpha) & \sin\theta\sin(\theta-\alpha) \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 \\ e^{-i\Gamma}\sin\alpha & 0 \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta\cos(\theta-\alpha)\cos\alpha - \cos\theta\sin(\theta-\alpha)e^{-i\Gamma}\sin\alpha & 0 \\ \sin\theta\cos(\theta-\alpha)\cos\alpha - \sin\theta\sin(\theta-\alpha)e^{-i\Gamma}\sin\alpha & 0 \end{bmatrix} \\
&= (\cos(\theta-\alpha)\cos\alpha - \sin(\theta-\alpha)e^{-i\Gamma}\sin\alpha) \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix} \\
H &= (\cos(\alpha-\theta)\cos\alpha + \sin(\alpha-\theta)e^{-i\Gamma}\sin\alpha) \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix} && (18)
\end{aligned}
$$

The vector for the transmitted beam is given by

$$\overrightarrow{J_2} = H\overrightarrow{J_1}$$

Where

$$\overrightarrow{J_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

So

$$\begin{aligned}\overrightarrow{J_2} &= (\cos(\alpha-\theta)\cos\alpha + \sin(\alpha-\theta)e^{-i\Gamma}\sin\alpha)\begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \text{from (18)} \\ &= (\cos(\alpha-\theta)\cos\alpha + \sin(\alpha-\theta)e^{-i\Gamma}\sin\alpha)\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \qquad (19)\end{aligned}$$

The intensity of incident beam characterized electric field $\overrightarrow{E_o}$ is given by

$$I_o = \frac{\overrightarrow{E_o}\cdot\overrightarrow{E_o^\star}}{2} = \frac{E_o^2}{2} \qquad \text{as } \overrightarrow{E_o} = E_o\hat{x}$$

Where E_o is the magnitude of magnitude of $\overrightarrow{E_o}$ Since we have used normalized Jones Vector so the electric field $\overrightarrow{E}$ of transmitted beam is

$$\overrightarrow{E} = E_o\overrightarrow{J}$$

The transmitted intensity of the system is given by

$$\begin{aligned}I &= \frac{\overrightarrow{E}\cdot\overrightarrow{E^\star}}{2} \\ &= \frac{E_o\overrightarrow{J}\cdot E_o\overrightarrow{J^\star}}{2} \\ &= I_o\overrightarrow{J_2}\cdot\overrightarrow{J_2^*} \\ &= I_o(\cos(\alpha-\theta)\cos\alpha + \sin(\alpha-\theta)e^{-i\Gamma}\sin\alpha)(\cos\theta\hat{x} + \sin\theta\hat{y}) \\ &\cdot(\cos(\alpha-\theta)\cos\alpha + \sin(\alpha-\theta)e^{i\Gamma}\sin\alpha)(\cos\theta\hat{x} + \sin\theta\hat{y}) \\ &= I_o(\cos(\alpha-\theta)\cos\alpha + \sin(\alpha-\theta)e^{-i\Gamma}\sin\alpha)(\cos(\alpha-\theta)\cos\alpha + \sin(\alpha-\theta)e^{i\Gamma}\sin\alpha) \\ &(\cos\theta\hat{x} + \sin\theta\hat{y})\cdot(\cos\theta\hat{x} + \sin\theta\hat{y}) \\ &= I_o(\cos^2\alpha\cos^2(\alpha-\theta) + \sin^2\alpha\sin^2(\alpha-\theta) + \sin\alpha\cos\alpha\sin(\alpha-\theta)\cos(\alpha-\theta)(e^{-i\Gamma} + e^{i\Gamma}))(\cos^2\theta + \sin^2\theta) \\ &= I_o(\cos^2\alpha\cos^2(\alpha-\theta) + \sin^2\alpha\sin^2(\alpha-\theta) + \frac{\sin(2\alpha)\sin(2\alpha-2\theta)}{4}2\cos\Gamma) \\ &= I_o(\cos^2\alpha\cos^2(\alpha-\theta) + \sin^2\alpha\sin^2(\alpha-\theta) + \frac{\sin(2\alpha)\sin(2\alpha-2\theta)}{2}\cos\Gamma)\end{aligned}$$

Let

$$A = \cos^2\alpha\cos^2(\alpha-\theta) + \sin^2\alpha\sin^2(\alpha-\theta) \qquad (20)$$

and

$$B = \frac{\sin(2\alpha)\sin(2\alpha-2\theta)}{2}\cos\Gamma \qquad (21)$$

So

$$\begin{aligned}B &= \sin(2\alpha)\cos\Gamma\frac{\sin(2\alpha)\cos(2\theta) - \cos(2\alpha)\sin(2\theta)}{2} \\ &= \frac{\sin^2(2\alpha)\cos(2\theta)\cos(\Gamma)}{2} - \frac{\sin(2\alpha)\cos(2\alpha)\sin(2\theta)\cos(\Gamma)}{2} \\ &= \frac{\sin^2(2\alpha)\cos(2\theta)\cos\Gamma}{2} - \frac{sin(4\alpha)}{4}\sin(2\theta)(1-2sin^2(\frac{\Gamma}{2})\end{aligned}$$

$$= \frac{\sin^2(2\alpha)\cos\Gamma\cos(2\theta)}{2} + \frac{\sin(4\alpha)\sin(2\theta)\ \sin^2(\frac{\Gamma}{2})}{2} - \frac{\sin(4\alpha)\sin(2\theta)}{4} \tag{22}$$

$$\begin{aligned}
A &= \cos^2\alpha\cos^2(\alpha-\theta) + \sin^2\alpha\sin^2(\alpha-\theta)\\
&= \cos^2\alpha\cos^2(\alpha-\theta) + \sin^2\alpha(1 - cos^2(\alpha-\theta))\\
&= \cos^2\alpha\cos^2(\alpha-\theta) + \sin^2\alpha - \sin^2\alpha\cos^2(\alpha-\theta)\\
&= (\cos^2\alpha - \sin^2\alpha)\cos^2(\alpha-\theta) + \sin^2\alpha\\
&= \cos(2\alpha)(\cos\alpha\cos\theta + \sin\alpha\sin\theta)^2 + \sin^2\alpha\\
&= cos(2\alpha)(cos^2\alpha\cos^2\theta + \sin^2\alpha\sin^2\theta + 2\cos\alpha\cos\theta\sin\alpha\sin\theta) + \sin^2\alpha\\
&= \cos(2\alpha)(\cos^2\alpha\cos^2\theta + sin^2\alpha(1-\cos^2\theta) + \frac{\sin(2\alpha)\sin(2\theta)}{2}) + \sin^2(\alpha)\\
&= \cos(2\alpha)(\cos^2\theta(\cos^2\alpha - \sin^2\alpha) + \sin^2\alpha + \frac{\sin(2\alpha)\sin(2\theta)}{2}) + \sin^2(\alpha)\\
&= \cos(2\alpha)(\cos^2\theta\cos(2\alpha) + \sin^2\alpha + \frac{\sin(2\alpha)\sin(2\theta)}{2}) + \sin^2(\alpha)\\
&= \cos(2\alpha)(\cos(2\alpha)\frac{\cos(2\theta)+1}{2} + \sin^2\alpha + \frac{\sin(2\alpha)\sin(2\theta)}{2}) + \sin^2(\alpha)\\
&= \frac{\cos^2(2\alpha)\cos(2\theta)}{2} + \frac{\cos^2(2\alpha)}{2} + \frac{\cos(2\alpha)\sin(2\alpha)\sin(2\theta)}{2} + \cos(2\alpha)\sin^2\alpha + \sin^2\alpha\\
&= \frac{\cos^2(2\alpha)\cos(2\theta)}{2} + \frac{\cos^2(2\alpha)}{2} + \frac{\cos(2\alpha)\sin(2\alpha)\sin(2\theta)}{2} + \sin^2\alpha(1+\sin(2\alpha))\\
&= \frac{\cos^2(2\alpha)\cos(2\theta)}{2} + \frac{\cos^2(2\alpha)}{2} + \frac{\cos(2\alpha)\sin(2\alpha)\sin(2\theta)}{2} + \cos(2\alpha)\frac{1-\cos(2\alpha)}{2} + \sin^2\alpha\\
&= \frac{\cos^2(2\alpha)\cos(2\theta)}{2} + \frac{\cos^2(2\alpha)}{2} + \frac{\cos(2\alpha)\sin(2\alpha)\sin(2\theta)}{2} + \frac{\cos(2\alpha)}{2} - \frac{\cos^2(2\alpha)}{2} + \sin^2\alpha\\
&= \frac{\cos^2(2\alpha)\cos(2\theta)}{2} + \frac{\sin(4\alpha)\sin(2\theta)}{4} + \frac{1}{2} - \sin^2\alpha + \sin^2\alpha
\end{aligned}$$

$$= \frac{\cos^2(2\alpha)\cos(2\theta)}{2} + \frac{\sin(4\alpha)\sin(2\theta)}{4} + \frac{1}{2} \tag{23}$$

So,

$$\begin{aligned}
I &= I_o(A+B)\\
&= \frac{1}{2}I_o(1 + (\cos^2(2\alpha) + \cos\Gamma\sin^2(2\alpha))\cos(2\theta) + \sin^2(\frac{\Gamma}{2})\sin(4\alpha)\sin(2\theta))
\end{aligned}$$

Let y=I, $y_o = I_o$ and x=cos(2θ) then,

$$y = \frac{1}{2}y_o(1 + (\cos^2(2\alpha) + \cos\Gamma\sin^2(2\alpha))x + \sin^2(\frac{\Gamma}{2})\sin(4\alpha)\sqrt{1-x^2})$$

$$\Rightarrow 2(y - \frac{1}{2}y_o) = y_o((\cos^2(2\alpha) + \cos\Gamma\sin^2(2\alpha))x + \sin^2(\frac{\Gamma}{2})\sin(4\alpha)\sqrt{1-x^2})$$

$$2(y - \frac{1}{2}y_o) - y_o(\cos^2(2\alpha) + \cos\Gamma\sin^2(2\alpha))x = y_o(\sin^2(\frac{\Gamma}{2})\sin(4\alpha)\sqrt{1-x^2})$$

$$4(y - \frac{1}{2}y_o)^2 + y_o^2(\cos^2(2\alpha) + \cos\Gamma\sin^2(2\alpha))^2x^2 - 4(y - \frac{1}{2}y_o)y_o(\cos^2(2\alpha) + \cos\Gamma\sin^2(2\alpha))x$$

$$= y_o^2\sin^4(\frac{\Gamma}{2})\sin^2(4\alpha)(1-x^2)$$

$$y_o^2((\cos^2(2\alpha) + \cos(\Gamma)\sin^2(2\alpha)^2 + sin^4(\frac{\Gamma}{2})\sin^2(4\alpha)))x + 4(y - \frac{1}{2}y_o)^2 - 4(y - \frac{1}{2}y_o)y_o(\cos^2(2\alpha) + \cos\Gamma\sin^2(2\alpha))x$$

$$= y_o^2\sin^4(\frac{\Gamma}{2})\sin^2(4\alpha)$$

This is the equation of an ellipse.

If we substitute α by $\frac{\pi}{4}$ in our equation for transmitted intensity we get

$$I = \frac{1}{2} I_o (1 + \cos\Gamma \cos(2\theta))$$

So if Intensity is plotted against $\cos(2\theta)$ in this case a straight line will result. In this experiment we will verify these predictions of theory by plotting I and $\cos(2\theta)$ for some angles.

2.1 Experiment

In this experiment we used a red He-Ne Laser of wavelength 632 nm. The intensity of the output beam was registered by means of a photodiode as the voltage on CRO.

For every value of α used θ was varied in steps of $5°$ from 0 to $180°$.

Before inserting the waveplate, the relative angle between transmission axes of the polarizer and the analyzer was determined by adjusting for maximum transmission. The angle for maximum transmission was marked as $\theta = 0$ and all other measurements of θ were taken relative to it.

Then the waveplate was inserted in between polarizer and analyzer and the relative angle between transmission axis of polarizer and optic-axis of waveplate was determined by again adjusting for maximum transmission. The angle for maximum transmission was marked as $\alpha = 0$ and all measurements of α were taken relative to it.

Then we proceeded by choosing a particular value of α and varying the value of θ from 0 to $180°$ in steps of $5°$ for the same value of α. We repeated this produce for 4 different values of α.

3 Results

For $\alpha = 45°$ least square curve fitting yielded a least square curve fit yielded a straight line as predicted by the theory. The Slope of the line was 125mV and its Intercept was 116mV.

For $\alpha = 20°$,$35°$ and $60°$ the figures obtained were nearly elliptical for the plot of the output intensity vs $\cos(2\theta)$ which again is compatible with theoretical prediction.

3.1 Uncertainties

The uncertainties in this experiment are due to the least counts of CRO and dials on the polarizer,analyzer and quarter wave plate.

The least count of CRO mode used was ± 2.5 mV.

The least count of the dials was $\pm 1°$.

3.2 Possible causes for errors

The following factors caused inaccuracies in the obtained results

3.2.1 Ambient Light

The experiment was performed in the daylight and the ambient light was a source of error in the measurements

3.2.2 Limitations of CRO

The CRO has low accuracy specially when compared with devices using digital displays and cannot accurately determine small changes in voltages. This induces a basic limitations in measuring the output intensity.

3.2.3 Heating up of Laser

Heating up of the laser affects its performance by reducing its ability to keep its intensity constant

3.2.4 Imperfect Alignment

Imperfect alignment causes the path of light to deviate from that of normal incidence which can induce considerable errors.

4 Conclusion

The passing of light through the birefringent materials can introduce phase difference between components of light having different polarizations, due to which linear polarization of incident light beam changes to circular or elliptical polarizations. This effect can be analyzed theoretically using Jones calculus. As the previous pages show, the theoretical predictions match the experimental results to fair extent.

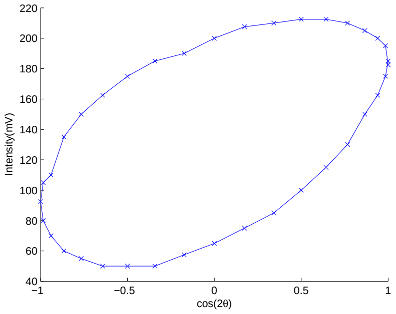
5 Data and Figures

$\theta(°)$	$\cos(2\theta)$	I(20°)(mV)	I(35°)(mV)	I(45°)(mV)	I(60°)(mV)
0	1	185	220	225	215
5	0.985	175	215	225	215
10	0.940	162.5	210	220	220
15	0.867	150	200	217.5	215
20	0.766	130	190	212.5	215
25	0.643	115	172.5	200	210
30	0.5	100	155	190	202.5
35	0.342	85	135	175	197.5
40	0.174	75	115	155	187.5
45	0	65	100	130	170
50	−0.174	57.5	72.5	110	155
55	−0.342	50	50	90	135
60	−0.5	50	40	70	115
65	−0.643	50	27.5	50	95
70	−0.766	55	20	32.5	80
75	−0.866	60	10	20	65
80	−0.940	70	10	10	47.5
85	−0.985	80	12.5	5	40
90	−1	92.5	20	2.5	30
95	−0.985	105	27.5	5	25
100	−0.940	110	40	10	20
105	−0.866	135	57.5	20	22.5
110	−0.766	150	80	35	27.5
115	−0.643	162.5	95	52.5	35
120	−0.5	175	112.5	70	45
125	−0.342	185	135	90	57.5
130	−0.174	190	155	110	72.5
135	0	200	175	135	90
140	0.174	207.5	190	155	110
145	0.342	210	200	172.5	130
150	0.5	212.5	210	190	150
155	0.643	212.5	215	200	170
160	0.766	210	220	207.5	180
165	0.866	205	22.5	215	190
170	0.940	200	220	220	200
175	0.985	195	220	222.5	205
180	1	182.5	217.5	222.5	210

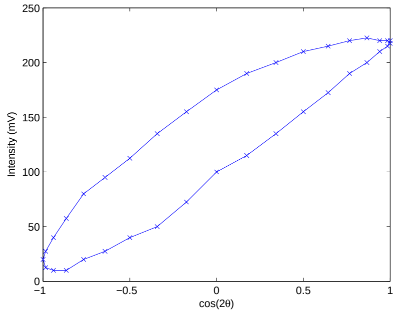
References

[1] Eugene Hecht, *Optics*. Addison Wesley, San Francisco, 4th Edition, 2002.

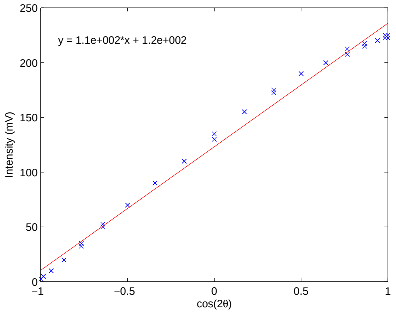
[2] Charles A. Bennet, *Optics*. Wiley, Chichester, 2008.



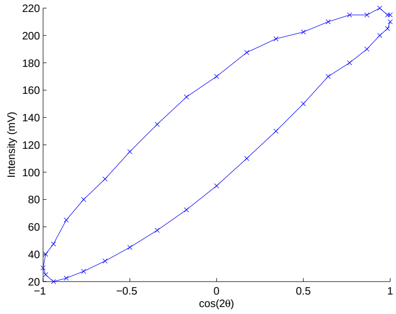
(a) $\alpha = 20°$



(b) $\alpha = 35°$

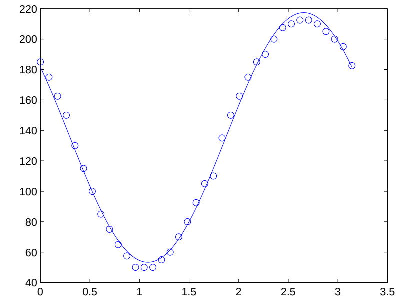


(c) $\alpha = 45°$

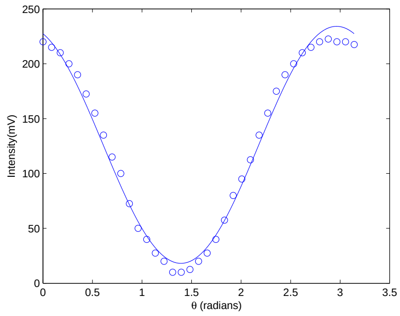


(d) $\alpha = 60°$

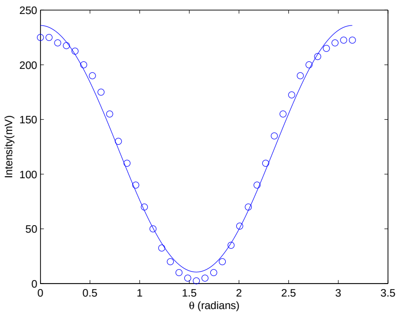
Figure 3: Graph of Intensity vs $\cos(2\theta)$ for different values of α



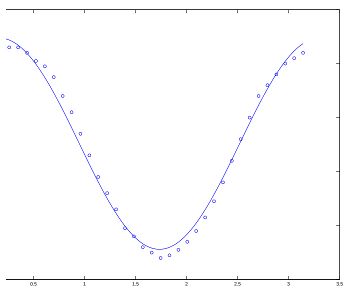
(a) $\alpha = 20°$



(b) $\alpha = 35°$



(c) $\alpha = 45°$



(d) $\alpha = 60°$

Figure 4: Graph of Intensity vs θ for different values of α