Experimental verification of Poiseuille's law through Siphon action

Ahmed Waqas Zubairi, Amrozia Shaheen and
Muhammad Sabieh Anwar

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This experiment demonstrates the flow of water through siphon action. A measurement on the flow velocity and viscosity of water is carried out to experimentally verify the Poiseuille's law. It also provides a practical understanding on how the liquids flow with or without any external applied force.

KEYWORDS

Pressure · Laminar Flow · Viscosity · Equation of continuity · Bernoulli's equation · Reynold's number · Poiseuille's law

APPROXIMATE PERFORMANCE TIME 4 hours

1 Conceptual Objectives

In this experiment, we will,

1. understand the basic principle of siphon flow,
2. learn how to apply the Poiseuille's law,
3. understand the Bernoulli's equation,
4. calculate the viscosity and flow rate of water, and,

5. understand how errors propagate from measured to inferred quantities.

2 Experimental Objectives

In this experiment, we emptied a filled cylinder through a siphon tube and observe how mass flow rate varies with height and length of the tube. This is a computer controlled experiment in which electronic mass balance and Labview software are the key sources of data acquiring. From the acquired data, students will determine the Reynolds number and the value of the viscosity and do error analysis.

3 Theoretical background

Matter is normally classified as solid, liquid or gas. The time interval during which a substance changes its shape in response to an external applied force determines whether it is solid, liquid or gas. A fluid is a collection of molecules held together by weak cohesive forces, usually liquids and gases are termed as fluids because they deform in response to external forces. The fluid mechanics deals with static and dynamic characteristics. Some general properties of fluid flow are,

1. **Steady or non-steady:** The flow of a fluid is described by pressure, density and flow velocity at every point of the fluid. If these variables are constant in time then the flow is steady.

2. **Compressible or incompressible:** If the density of a fluid remain constant, does not depend on x, y, z and t, then the flow is incompressible.

3. **Viscous or non-viscous:** Viscosity is the resistance in the fluid flow. When a fluid flows such that there is no energy dissipation through viscous forces, then it is non-viscous flow.

4. **Rotational or irrotational:** If any element of the fluid does not rotate about an axis through the center of mass of the element, then the flow is irrotational.

4 Pressure

Pressure is simply defined as the force acting on an area. If a body rests on the surface then the force acting on it and the exerting pressure will be its weight. Mathematically, it can be written as,

$$P = \frac{F}{A}. \tag{1}$$

In equilibrium condition every portion of the fluid is in equilibrium and the net force and the net torque is zero.

Consider a small segment of the fluid at a distance y above some reference level as shown in Figure (1a). This segment is a thin disk with thickness dy and area A. The mass of the element is $dm = \rho dV = \rho A\, dy$ and weight $W = (dm)g = \rho g A\, dy$. Since there is no acceleration along the horizontal direction as represented in Figure (1b, 1c), therefore the resultant horizontal force is zero and the net vertical force is,

$$\Sigma F_y = PA - (P + dP)A - \rho g A\, dy = 0, \tag{2}$$

yielding,

$$\frac{dP}{dy} = -\rho g. \tag{3}$$

The above equation tells the pressure variation with elevation above some reference level for a fluid in static equilibrium. As the height increases (dy positive), the pressure decreases (dP negative). For an incompressible and homogeneous liquid with difference in height, the pressure is,

$$P_2 - P_1 = -\rho g(y_2 - y_1), \tag{4}$$

and if the liquid has a free surface shown in Figure (1d), then,

$$P_o - P = -\rho g(y_2 - y_1), \tag{5}$$

$$P = P_o + \rho g h, \tag{6}$$

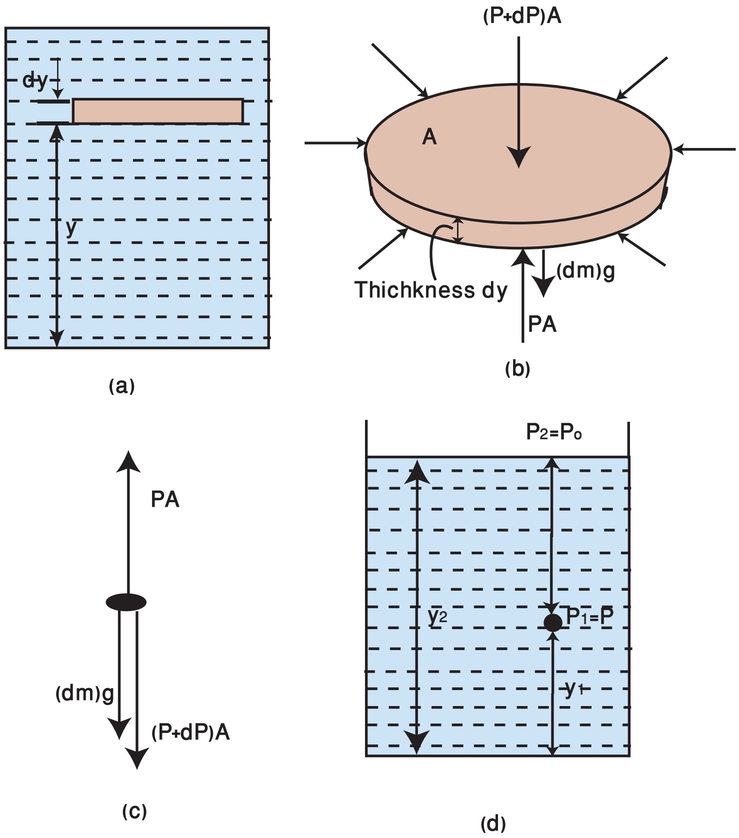


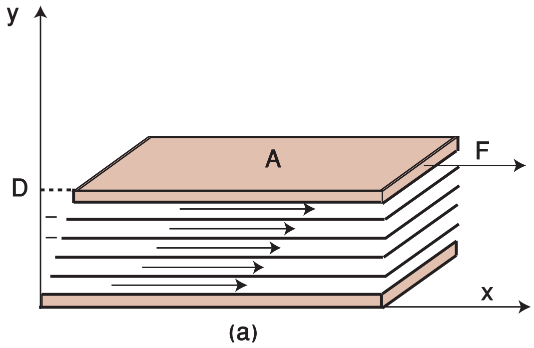
Figure 1: A static fluid. (a) Small element at rest, (b) forces acting on small element, (c) free body diagram, and (d) a fluid with top surface open to the atmosphere.

where, $y_2 - y_1 = h$. The above expression shows that the pressure in a liquid increases with depth but would be same at all those points that are on the same level.

4.1 Viscosity

When you pour a glass of juice, the liquid flows freely and quickly. But when you pour syrup on cakes, that liquid flows slowly and sticks to it. The difference is fluid friction both within the fluid itself and between the fluid and its surroundings. This property of fluids is called viscosity. In liquids, it originates with intermolecular cohesive forces and increase in kinetic energy of the molecules of the fluid weakens the effect of intramolecular forces.

Consider the motion of the fluid between two parallel plates as shown in



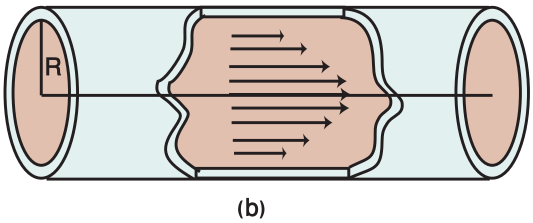


Figure 2: Fluid flow. (a) A fluid filling the space between two plated separated by a distance D, (b) fluid flowing through a cylindrical pipe with radius R.

Figure (2a). A force F is applied to the upper plate to move it with constant velocity v relative to the lower one plate. This force opposes the drag force. A fluid can be thought of consisting of number of layers parallel to the plates, each of thickness dy. The coefficient of viscosity is the ratio of stress to strain,

$$\eta = \frac{F/A}{dv/dy} = \frac{FD}{vA}. \tag{7}$$

Assuming the velocity of the bottom layer is zero and the velocity gradient is simply v/D, where D is the distance between the plates.

A practical example of viscosity is the fluid flow through the cylindrical pipes. The flow is laminar and each layer is a thin walled cylinder of different radii as shown in Figure (2b). The flow velocity varies with the radius, maximum on the radial axis and minimum (almost zero) at the walls. The total volume flux through each cylindrical shell is given by Poiseuille's law,

$$\frac{dV}{dt} = \frac{\pi R^4 \Delta P}{8\eta L}. \tag{8}$$

where R is the radius and L is the length of the tube, η is the viscosity of the fluid and ΔP is the change in pressure.

Q 1. Why do auto manufacturers recommend using 'multi viscosity' engine oil in cold weather?

Q 2. Liquid Mercury (viscosity=1.55×10^{-3} N.s/m^2) flows through a horizontal pipe of internal radius 1.88 cm and length 1.26 m. The volume flux is 5.35×10^{-2} l/min.

(a) Show that the flow is laminar.

(b) Calculate the difference in pressure between the two ends of the pipe.

4.2 Bernoulli's equation

When a fluid moves through a region in which either the speed of the fluid or elevation above the Earth's surface changes, the impact is that the pressure in the fluid changes. The relationship between fluid speed, pressure and elevation was first derived by Daniel Bernoulli in 1738. Bernoulli's equation, a fundamental relation in fluid mechanics is derivable from the basic laws of Newtonian mechanics.

Consider a steady, incompressible and nonviscous flow of a fluid through a pipeline from position shown in Figure (a) to (b). The portion at the left has a uniform cross sectional area A_1 and at an elevation y_1 from some reference level. It gradually rises and after time Δt, the portion moves to the right with cross section area of A_2 and at an elevation of y_2.

According to work-energy theorem, the work done by the resultant force that acts on a system is equal to the change in kinetic energy. Assuming that there is no viscous force, the only forces that do work on the system are the pressure forces and the force of gravity. The net work done on the system by all the forces is,

$$W = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - \Delta m \, g(y_2 - y_1). \tag{9}$$

The pressure force $P_2 A_2 \Delta l_2$ is negative because its direction is opposite to the horizontal displacement. The gravitational force is also negative

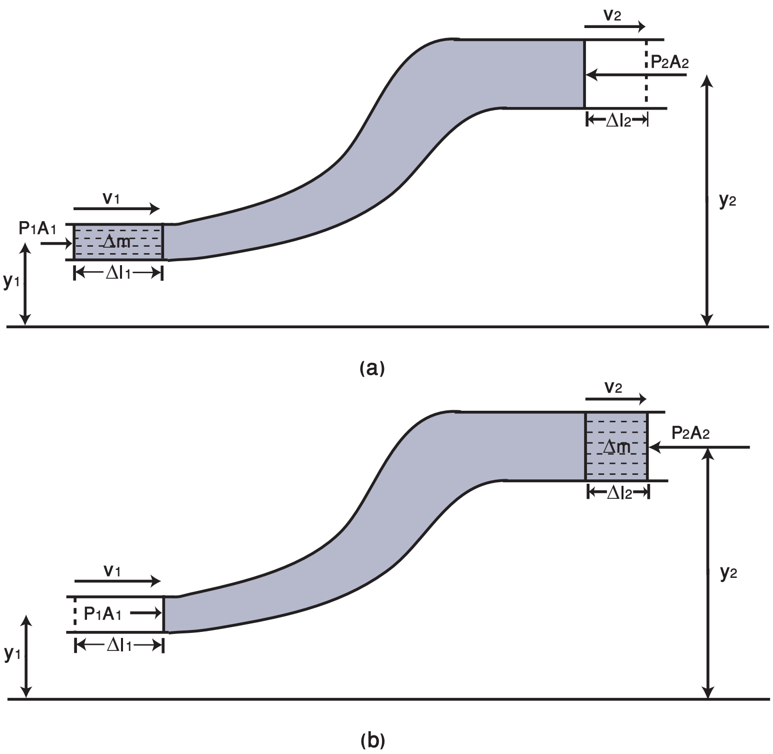


Figure 3: A fluid is flowing through a pipe from position (a) to (b). The net effect is the transfer of the element (Δm) from left to the right end.

because it acts in a direction opposite to the vertical displacement. As ($A_1 \Delta l_1 = A_2 \Delta l_2$) is the volume of the fluid (ΔV), we can write it as $\Delta m/\rho$. The change in kinetic energy is,

$$\Delta K \;=\; \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2. \tag{10}$$

By equating Equations (9) and (10), we get,

$$(P_1 - P_2)(\Delta m/\rho) - \Delta m\, g(y_2 - y_1) \;=\; \frac{1}{2}\Delta m\, v_2^2 - \frac{1}{2}\,\Delta m\, v_1^2,$$

and finally, the above expression becomes,

$$P_1 + \frac{1}{2}\,\rho\, v_1^2 + \rho\, g\, y_1 \;=\; P_2 + \frac{1}{2}\,\rho\, v_2^2 + \rho\, g\, y_2. \tag{11}$$

The above equation is often expressed as,

$$P + \frac{1}{2}\,\rho\, v^2 + \rho g y \;=\; \text{constant}\,. \tag{12}$$

The Equation (11) can be modified in many different ways depending upon the situation. If the fluid is at rest i.e. $v_1 = v_2 = 0$ then,

$$P_1 + \rho g h_1 = P_2 + \rho g h_2, \tag{13}$$

where the term $(P + \rho g y)$ is called the *static pressure*.

Now, if the both ends of the pipe are placed at same height then Equation (11) can be re-written as,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2, \tag{14}$$

which means that large speed corresponds to low pressure. Whereas, the term $\frac{1}{2}\rho v^2$ is called the *dynamic pressure*.

Q 3. A giraffe needs a strong heart because of its long neck. Suppose the difference of height between the aortic valve (the place where the arterial blood comes out of the heart) and the head of a giraffe is 2.50 m, and the artery leading from near the aortic valve to the head has constant cross section all the way to the head. Blood is an incompressible fluid with density $1.0\,\mathrm{g/cm^3}$. Assume the pressure at the head is zero [3].

(a) What is the minimum pressure at the aortic valve? Compare this pressure to the peak output pressure of the human heart ($1.6 \times 10^4\,\mathrm{Pa}$)?

(b) What would be the effect on the giraffe if the artery diameter narrowed down as it approached the brain?

Q 4. Consider a uniform U-tube with a diaphragm at the bottom and filled with a liquid to different heights in each arm as shown in Figure (4a). Now imagine that the diaphragm is punctured so that the liquid flows from left to right [2].

(a) Show that the application of Bernoulli's equation to points 1 and 3 leads to a contradiction.

(b) Explain why Bernoulli's equation is not applicable here (Hint: Is the flow steady?).

Q 5. A tank of cross sectional area $0.07\,\mathrm{m^2}$ is filled with water as shown in Figure (4b). A tightly fitting piston with a total mass 10 kg rests on top of the water. A circular hole of diameter 1.5 cm is opened at the depth of 60 cm below the water level of the tank. What is the initial rate of flow of water out of the hole [3]?

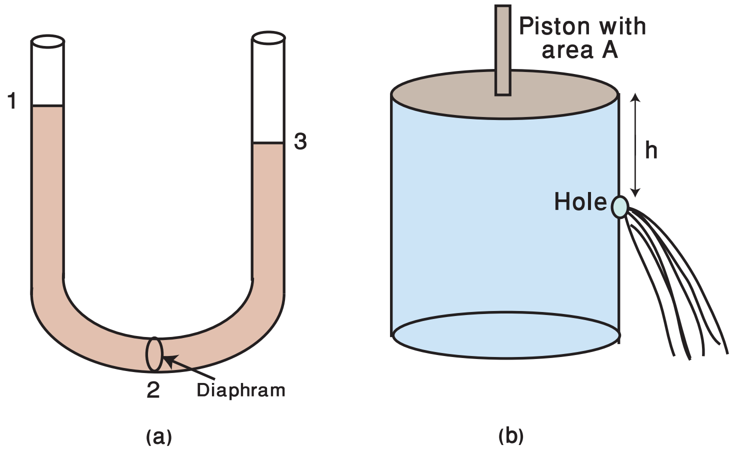


Figure 4: (a) A uniform u-tube with a diaphragm inside. (b) A water tank filled with water.

4.3 The siphon

The siphon is a device used for transferring liquid from a higher to a lower level. A typical siphon consists of a cylinder with a siphon tube immersed in it as shown in Figure (5). The working of a siphon is somewhat confusing if the fluid flows due to atmospheric pressure or it is the cohesion of the fluid [4]. Now let's try to make this thing clear through the following analysis,

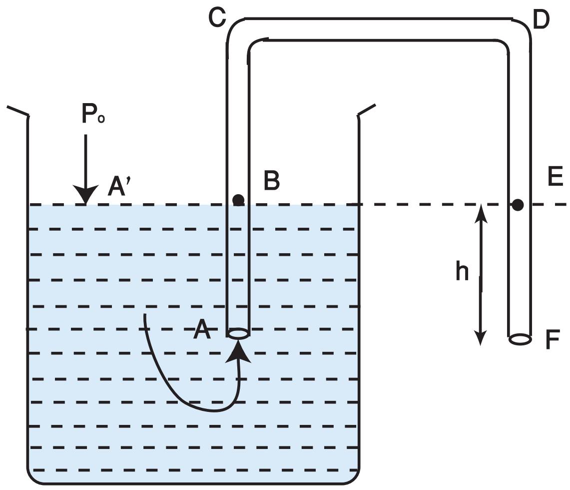


Figure 5: A schematic diagram of a siphon.

4.3.1 Hydrostatic model:

Consider points B and E inside the siphon, the pressure at these points is,

$$P_B = P_E = P_o \,(\text{atmospheric pressure}). \tag{15}$$

These points are gravitational iso-potential. The pressure at point F is,

$$P_E + \rho\, g\, h = P_o + \rho\, g\, h. \tag{16}$$

Now there are two options that the liquid flows out of the siphon tube:

1. If $P_F > P_o$,
2. if $P_B = P_o$, $P_F = P_o$, and $P_C > P_D$.

The first option is correct in static condition while the second one is incorrect.

As we are considering the flow of the liquid, hence we must have to apply the dynamic model.

4.3.2 Dynamic model

Suppose that the fluid is flowing through the siphon tube which is of uniform cross section area (A) as shown in Figure (6), then the mass per unit time through A, B, C is the same,

$$\rho A v = \text{constant} \;\; (\text{for incompressible fluids }), \tag{17}$$

where v is the velocity and ρ is density of the fluid. The above expression is called the equation of continuity.

Now let's assume that the flow of a fluid through the siphon tube is steady and no heat exchange is possible, then the Bernoulli equation is,

$$P + \frac{1}{2}\rho\, v^2 + \rho g h = \text{constant}. \tag{18}$$

As the surface area of the cylinder is quite large therefore the downward velocity of the fluid is very small (almost negligible). Consider points A' and B, the Bernoulli's equation becomes,

$$P_{A'} + \cancelto{0}{\frac{1}{2}\rho\, v_{A'}^2} + \rho g h = P_B + \frac{1}{2}\rho\, v_B^2 + \rho g h,$$

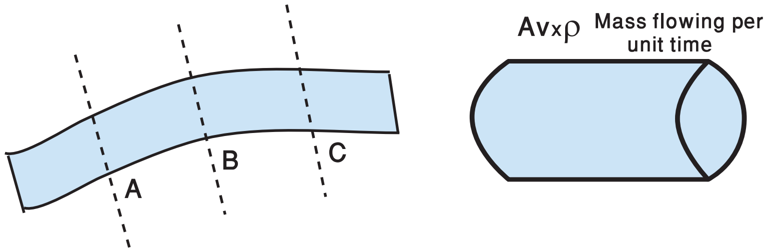


Figure 6: A schematic diagram of a siphon.

where $P_{A'} = P_o$, and $v_B = v$ and the above expression becomes,

$$P_o = P_B + \frac{1}{2}\rho v^2. \tag{19}$$

Similarly, at points B & F,

$$P_B + \frac{1}{2}\rho v^2 + \rho g h \;=\; P_F + \frac{1}{2}\rho v^2, \tag{20}$$

here $P_F = P_o$. Substituting Equation (19) in (20), we get,

$$v^2 \;=\; 2gh. \tag{21}$$

The above expression is similar to the velocity of a solid mass falling through a height h. In dynamic situations, pressures are less than in the static situation by a factor of $\frac{1}{2}\rho v^2$.

Q 6. Write down Bernoulli's equations for points (A' and F), (Q and F), (B and E) and (C and F). Find the pressure values at all these points.

Reynold's number: The Reynolds number is a non-dimensional quantity that describes the type of the flow either laminar or turbulent. Mathematically, it has the following form,

$$R \;=\; \frac{v d \rho}{\eta}, \tag{22}$$

where v is the velocity of the flow, d is diameter of the pipe, ρ is the density and η is the viscosity of the fluid. The flow can be distinguished as,

1. Laminar, if $R < 2000$,

2. Turbulent, if $R > 2000$.

Laminar flow by including the effect of friction: Both laminar and turbulent flow depend on the velocity that does not remain constant due to energy losses for any viscous fluid and it is inappropriate to use Bernoulli's equation. The Bernoulli's equation can be applicable to any viscous fluid if we account for friction. The average velocity becomes,

$$v^2 = 2g(h - h_L). \tag{23}$$

where h_L is the loss of head due to skin friction in laminar flow. It is given by Darcy's equation,

$$h_f = f\frac{l}{d}\frac{v^2}{2g}. \tag{24}$$

According to Poiseuille equation, the average velocity is,

$$\begin{aligned} v &= \frac{d^2}{32\eta} \times \left(\frac{\Delta P}{l}\right), \\ &= \frac{d^2}{32\eta} \times \left(\frac{\rho g h - \frac{1}{2}\rho v^2}{l}\right). \end{aligned} \tag{25}$$

Multiplying by v and substituting Equation (22) in the above expression, we get,

$$\begin{aligned} v^2 &= (R)\frac{d^2}{32\eta}\left(gh - \frac{1}{2}v^2\right), \\ v &= \left(\frac{2g}{1 + (\frac{64}{R})\frac{l}{d}}\right)^{1/2} \sqrt{h}. \end{aligned} \tag{26}$$

Now,

$$v = \left(\frac{2g}{1 + (\frac{64}{R})\frac{l}{d}}\right)^{1/2} \sqrt{h} = -\left(\frac{dh}{dt}\right). \tag{27}$$

Rewriting the above expression,

$$\frac{dh}{dt} = -C(h^{1/2}), \tag{28}$$

where $C = \sqrt{\frac{2g}{1+(\frac{64}{R})\frac{l}{d}}}$ is a constant.
Integrating the above equation yields,

$$\sqrt{h} = -\frac{Ct}{2} + k. \tag{29}$$

Q 7. Derive Equation (29). (Hint: Substitute $\sqrt{h} = z$).

5 The Experiment

5.1 Apparatus

1. Two measuring cylinders of same geometry,
2. Siphon tube,
3. Electronic mass balance,
4. Height adjuster stand,
5. Stands and corks for tube fittings,
6. Meter rule, and,
7. Vernier calliper.

5.2 Experimental procedure

Complete the assembly as shown in Figure (). You are provided with two cylinders of same geometry and a siphon tube. Adjust the position of the provided siphon tube in an inverted u-shape configuration using stand and cork. Place cylinder 1 on electronic mass balance and set it to zero in order to subtract the mass of the cylinder from the data. Place both cylinders at some different height.

★ **Q** 8. Measure the height difference between two cylinders using meter rule.

★ **Q** 9. Measure the length L and radius r of the siphon tube using vernier calliper. Measure the radius R of the cylinder.

★ **Q** 10. Note down uncertainties in your measurements.

★ **Q** 11. Run the Labview file **mass record of emptying cylinder.vi** by clicking the arrow on the left side of the file. When you run this file, you are asked to enter the file name. Enter any name and save it in your z drive.

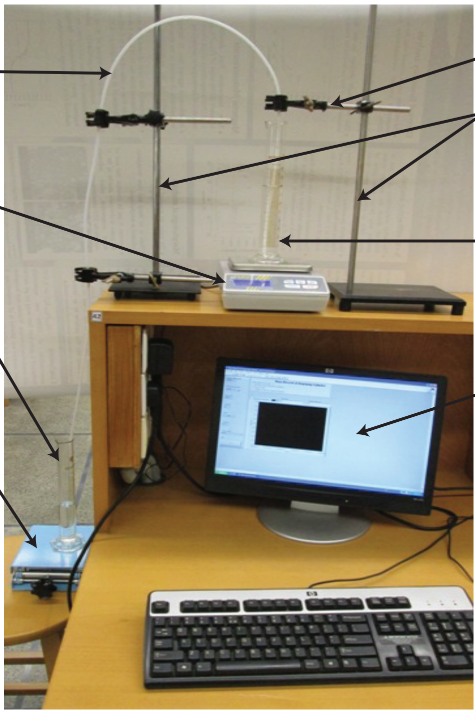


Figure 7: The experimental scheme.

★ **Q 12.** Pour water in cylinder 1 that is being placed on the electronic mass balance and suck water through the siphon tube such that it completely fills up with water. Make sure no bubbling is present inside the siphon tube because it would affect the velocity of the fluid inside the siphon tube.

★ **Q 13.** Press **Enter** and let the water flow through the siphon tube. Make sure both steps must be done simultaneously and no time delay is present between the two processes.

★ **Q 14.** Your data is saved in an **lvm** file. Open Matlab and change its directory. Make sure the data file must be in the current directory of the Matlab. Load the data file using the command (**a=load('filename.lvm')**). Extract columns of time and mass (time=a(:,1), mass=a(:,2)).

★ **Q 15.** You are provided with an M-file. Add this file to the current directory of the Matlab and run it, you will get the vectors for height h and time t.

★ **Q 16.** Plot a graph of h versus t^2 and use Equation (29) to calculate the value of the viscosity ($\rho_{\text{water}} = 1.0 \times 10^3\,\text{Kg/m}^3$).

★ **Q 17.** Calculate uncertainty in the measured value of viscosity using weighted fit of a straight line.

★ **Q 18.** Suggest two improvements in order to get more accurate value of the viscosity.

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