Second recitation on statistical mechanics

1. A nanoparticle containing 6 atoms can be modeled approximately as an Einstein solid of 18 independent oscillators. The evenly spaced energy levels of each oscillator are $4 \times 10^{-21}$ J apart.

(a) When the nanoparticle’s energy is in the range $5 \times 4 \times 10^{-21}$ J to $6 \times 4 \times 10^{-21}$ J, what is the approximate temperature? (In order to keep precision for calculating the heat capacity, give the result to the nearest tenth of a Kelvin.)

(b) When the nanoparticle’s energy is in the range $8 \times 4 \times 10^{-21}$ J to $9 \times 4 \times 10^{-21}$ J, what is the approximate temperature? (In order to keep precision for calculating the heat capacity, give the result to the nearest tenth of a degree.)

(c) When the nanoparticle’s energy is in the range $5 \times 4 \times 10^{-21}$ J to $9 \times 4 \times 10^{-21}$ J, what is the approximate heat capacity per atom? For your convenience, the entropy-energy graph is also shown.

![Entropy-Energy Graph](image)

2. For a certain metal the stiffness of the interatomic bond and the mass of one atom are such that the spacing of the quantum oscillator energy levels is $1.5 \times 10^{-23}$ J. A nanoparticle of this metal consisting of 10 atoms has a total thermal energy of $18 \times 10^{-23}$ J.

Assume all the internal energy is of the disordered kind. (a) What is the entropy of
this nanoparticle?

(b) The temperature of the nanoparticle is 87 K. Next we add $18 \times 10^{-23}$ J to the nanoparticle. By how much does the entropy increase?

3. A 50 gram block of copper (one mole has a mass of 63.5 grams) at a temperature of $35^\circ$ C is put in contact with a 100 gram block of aluminum (molar mass 27 grams) at a temperature of $20^\circ$ C. The blocks are inside an insulated enclosure, with little contact with the walls. At these temperatures, the high temperature limit is valid for the specific heat capacity, $C_v = 3k_B$. Calculate the final temperature of the two blocks. Do NOT look up the specific heat capacities of aluminum and copper; these have been provided to you.