

Analyzing Simple Pendulum Phenomenon

Bilal Aftab Usman and Muhammad Sabieh Anwar

Syed Babar Ali School of Science and Engineering, LUMS

Version 1; July 4, 2014

The motion of the simple pendulum has been studied for centuries. The most common outcome of this classic experiment is to determine time period of the oscillation or the value of acceleration due to gravity. In this experiment, we will go a step further to study and analyse the underlying physical quantities like angular displacement, velocities and acceleration. The results will then be verified by correlating with acceleration data obtained from the smartphone.

Essential pre-lab reading: “*Physics for Engineers and Scientists : Extended 3rd Edition*” by Ohanian, H.C. and Markert, J.T. (W.W. Norton and Company, 2007); (Sections 12.2-12.4 and 15.1-15.4)

1 Test your understanding

1. Derive the transformation from cartesian coordinates to radial coordinates, and vice versa.
2. What is the period of oscillation of a simple pendulum?

2 The Experiment

We will use the smartphone as the bob of a simple pendulum recording accelerations through the built-in accelerometer while we will record the oscillatory motion in the lab (Cartesian) frame using a digital video camera and use it for quantitative analysis. In the end, we will correlate the results obtained through both methods.

Start by attaching the smartphone to the specially prepared base using cable-ties. Now, place a bulls eye level on top of the base and add counterweights as shown in figure (1d) until the whole assembly is level. This will serve as the pendulum bob. Now, mount the digital camera on the mini-tripod and turn it on. Adjust the frame so that the bob assembly is visible in the centre of the frame (use figure (1b) as reference). For this experiment, we will record at 120 fps.

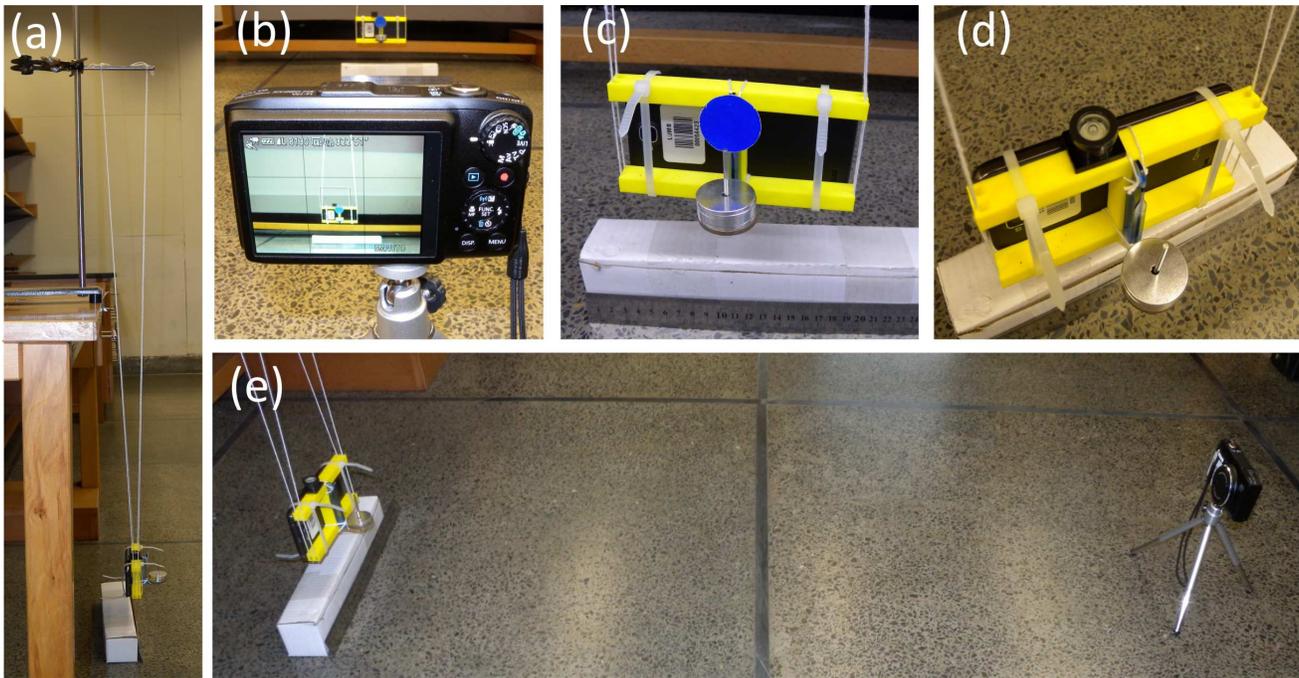


Figure 1: Experimental Setup. (a) Side view of the pendulum, (b) sample frame, (c) attaching smartphone with cable-ties and placement of the marker, (d) balancing the bob, (e) camera on a tripod placed about 5-6 feet from the plane of oscillation.

Now, launch the AndroSensor app on the smartphone and set the accelerometer to **START** recording. Now, hold the bob assembly at a small displacement of about 10-15 cm from the mean position and release. Allow the oscillatory motion to stabilise and then press the **RECORD** button on the camera. Record about 4-5 complete cycles and then press the record button on the camera again to **STOP** recording. After this, immediately **STOP** recording on the smartphone accelerometer.

A single video along with the corresponding accelerations should be enough for quantitative analysis but you can repeat as many times as you wish to obtain a few datasets.

3 Analysis

3.1 Preparing Data for Quantitative Analysis

Make sure the video is in the same folder and then run the MATLAB function “**trackmob.m**”. This function automatically tracks the position of the marker returns the time and tracked position data in cartesian coordinates. The syntax for using this function is “[*time, position*]=**trackmob**(‘*filename.mov*’, *knownDistance*);”. Identify two points of known distance (in metres) on the first frame using the visible crosshairs. Make sure that the value of *knownDistance* is correct and in metres.

Q 1. While the tracking script is running, try to guess the type of the two position datasets.

What result do you expect?

When you obtain the results, plot the position data.

Q 2. What do the two datasets you have obtained represent?

Q 3. Do you notice anything unexpected in either of the plotted datasets? If yes, can you give a reason for this unexpected result.

Now, we have datasets for position of the bob in lab frame. We can move on to the quantitative analysis. You are free to use your own variable names. However, the following scheme suggested in Table (1) is recommended to avoid confusion and keeping track of your data.

Physical Quantity	Variable Name
x-coordinate of tracked data	<i>position(:,1)</i>
y-coordinate of tracked data	<i>position(:,2)</i>
time	<i>time</i>
center of the circle / pivot point	<i>(xc,yc)</i>
radius of the circle / length of pendulum	<i>R</i>
angular displacement θ	<i>theta</i>
angular velocity ω_v	<i>thetadot</i>
angular acceleration α	<i>thetaddot</i>
time for angular velocity and angular acceleration	<i>timed</i>
centripetal acceleration a_c	<i>ac</i>
tangential acceleration a_t	<i>at</i>

Table 1: Suggested variable names for the experiment.

3.2 Video Analysis

We will now transform the datasets available in Cartesian coordinates to the radial coordinates. But first, we will fit the obtained position data to a circle. We know that the oscillation effectively represents motion on a small arc of a circle. We will fit our datapoints to this arc and obtain the radius of the circular motion. Use the function “**circlefit.m**” (Syntax: “[*xc, yc, R*] = *circlefit(x, y);*”) to fit the x and y datapoints to the circle. The function gives the radius and the coordinates of the centre of the circle as output.

Q 4. What do the outputs xc and yc represent in terms of our simple pendulum experiment?

Q 5. What does this radius represent in terms of our simple pendulum experiment?

Now, we will subtract the x -coordinates from the mean position of x -datapoints and similarly subtract the y -coordinates from the mean position of y -datapoints.

Q 6. Why have we translated the datapoints? (Do not spend too much time on this question

and carry on with the experiment if you're stuck.)

Q 7. Why do we need to transform the data into radial coordinates?

We will now proceed with the transformation of coordinates. You already have the first radial coordinate, which is the radius R . Use the following transformation to obtain an array for the other radial coordinate θ ,

$$\theta = \arctan\left(\frac{x}{R-y}\right),; \quad (1)$$

where x and y are now the rescaled coordinates.

Plot the theta coordinate with respect to time and try to determine the time period of one complete oscillation.

Q 8. What does θ represent in terms of our simple pendulum experiment? (Hint: This is the analog of linear displacement in circular motion.)

Q 9. What are the maximum and minimum values of θ ? Are they equal in magnitude?

Now, we expect θ to vary like a simple harmonic oscillator because the pendulum is swinging through very small angles. We will use the following model function for a damped simple harmonic oscillator,

$$f(t) = x_o \cos(\omega t + \phi) \exp(\gamma t) + b. \quad (2)$$

You will fit the dataset to this model using the least-squares fitting function “**lsqfun.m**”. The syntax for using this function is “ $c = \mathbf{lsqfun}(xdata,ydata);$ ”. The variable “ c ” stores the five unknown parameters in the model described above.

Q 10. What does γ represent in equation (2)?

Q 11. Use the model function above and the values of the five parameters to interpolate the time and position data by a factor of 100.

Q 12. Why do we interpolate the time and position data by a factor of 100?

You will now differentiate the interpolated time series for θ to obtain angular velocity ω_v and angular acceleration α . Use the “**deriv.m**” function for differentiation (Syntax: “[$xdata,ydata$] = **deriv**($xdata,ydata,order$);” where order is 1 for computing a first-order derivative and 2 for a second-order derivative.)

Q 13. Compute the first and second order derivatives for θ . Plot your results to obtain graphs for ω_v and α .

Q 14. What is the difference between ω in equation (2) and ω_v ? Or do they represent the same physical quantity?

We can now use the values of angular velocity ω_v and angular acceleration α to calculate the centripetal acceleration a_c and the tangential acceleration a_t respectively.

Q 15. Calculate and plot the measured a_c and a_t .

3.3 Correlation with Accelerometer data

We will now compare the centripetal acceleration and the tangential acceleration with the acceleration data obtained from the accelerometer. But first, we need to determine which video analysis dataset needs to be correlated with the accelerometer data. The accelerometer has a fixed coordinate system which is obviously rotating when the pendulum is swinging.

Q 16. Along which fixed coordinate (of the smartphone) will you obtain the value of centripetal acceleration?

Q 17. Along which fixed coordinate (of the smartphone) will you obtain the value of tangential acceleration?

Now, plot the linear accelerations determined in the previous question and compare them with the plots obtained with video analysis.

Q 18. Comment on the correlation of the results obtained separately through both methods of data collection.