9.8.3 The Rotating Sagnac Interferometer

The Sagnac Interferometer is widely used to measure rotational speed. In particular, the ***ring laser***, which is essentially a Sagnac Interferometer containing a laser in one or more of its arms, was designed specifically for that purpose. The first ring laser gyroscope was introduced in 1963, and work is continuing on various devices of this sort (see photo). The initial experiments that gave impetus to these efforts were performed by Sagnac in 1911. At that time he rotated the entire interferometer, mirrors, source, and detector, about a perpendicular axis passing through its center (Fig. 9.56). Recall, from Section 9.4.2, that two overlapping beams traverse the interferometer, one clockwise, the other counterclockwise. The rotation effectively shortens the path taken by one beam in comparison to that of the other. In the interferometer, the result is a fringe shift proportional to the angular speed of rotation ω. In the ring laser, it is a frequency difference between the two beams that is proportional to ω.

Consider the arrangement depicted in Fig. 9.56. The corner A (and every other corner) moves with a linear speed $v = R\omega$, where R is half the diagonal of the square. Using classical reasoning, we find that the time of travel of light along AB is

$$t_{AB} = \frac{R\sqrt{2}}{c - v/\sqrt{2}}$$

An early ring laser gyro. (Photo courtesy Autonetics, a Division of Boeing North America, Inc.)

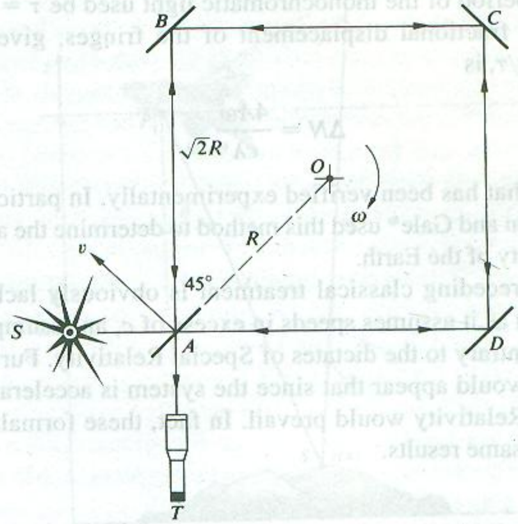

Figure 9.56 The rotating Sagnac Interferometer. Originally it was 1 m × 1 m with ω = 120 rev/min.

or

$$t_{AB} = \frac{2R}{\sqrt{2}c - \omega R}$$

The time of travel of the light from A to D is

$$t_{AD} = \frac{2R}{\sqrt{2}c + \omega R}$$

The total time for counterclockwise and clockwise travel is given respectively by

$$t_{\circlearrowleft} = \frac{8R}{\sqrt{2}c + \omega R}$$

and

$$t_{\circlearrowright} = \frac{8R}{\sqrt{2}c - \omega R}$$

For $\omega R << c$ the difference between these two intervals is

$$\Delta t = t_{\circlearrowright} - t_{\circlearrowleft}$$

or, using the Binomial Series,

$$\Delta t = \frac{8R^2\omega}{c^2}$$

This can be expressed in terms of area $A = 2R^2$ of the square formed by the beams of light as

$$\Delta t = \frac{4A\omega}{c^2}$$

Let the period of the monochromatic light used be $\tau = \lambda/c$; then the fractional displacement of the fringes, given by $\Delta N = \Delta t/\tau$, is

$$\Delta N = \frac{4A\omega}{c\lambda}$$

a result that has been verified experimentally. In particular, Michelson and Gale* used this method to determine the angular velocity of the Earth.

The preceding classical treatment is obviously lacking, inasmuch as it assumes speeds in excess of c, an assumption that is contrary to the dictates of Special Relativity. Furthermore, it would appear that since the system is accelerating, General Relativity would prevail. In fact, these formalisms yield the same results.

9.8.4 Radar Interferometry

In February 2000 the Space Shuttle Endeavour completed a mission to create a "three-dimensional" map of the Earth covering 119 million square kilometers. The feat was accomplished using synthetic aperture radar (SAR). In general, the larger the aperture of a viewing system, the greater the resolution (p. 471) and the more details one can see. SAR is a technique for using the motion of an airplane or spacecraft along with signal processing methods to simulate a large antenna.

Using a phased array antenna (p. 98), the Shuttle swept a radar beam back and forth perpendicular to its line of motion painting a 225-km wide swath over the Earth's surface (Fig. 9.57). Orbiting upside-down, Endeavour extended a 60-m mast with two receiving antennas at its end (Fig. 9.58). The SAR then sent out a stream of about 1700 high-powered electromagnetic pulses per second from its main antenna in the cargo bay, which was both a transmitter and receiver. Actually, the mission utilized two different radars: a C-band system operating at a wavelength of 5.6 cm that provided most of the coverage, and a higher resolution X-band 3-cm system that gave a detailed view of a narrow 50-km swath (Fig. 9.57). A radar image is made up of countless tiny uniform dots known as pixels (p. 473). The pixel is the smallest bit of information in the picture—nothing can be seen that's smaller than a single pixel. For the main C-band system, each pixel is about 12.5 m in diameter, and the smallest object that can be resolved is about 30 m across.

*Michelson and Gale, *Astrophys. J.* **61**, 140 (1925).

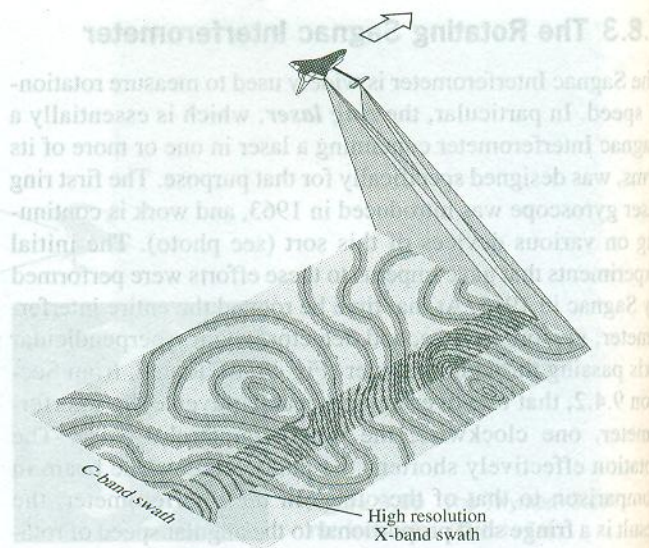

Figure 9.57 As the Shuttle orbited, its two radar systems swept out a swath across the surface of the Earth.

Ordinarily, a radar system sends out a pulse (with a pulsewidth of 10-50 μs), and then, picking up the backscattered wave, it records both the amplitude and round-trip time. That gives a rough idea of the size and location of the target. However, in order to gather data about the elevation of surface features on the Earth, the Shuttle Radar Topography Mission (SRTM) utilized interferometry—in a way that suggests

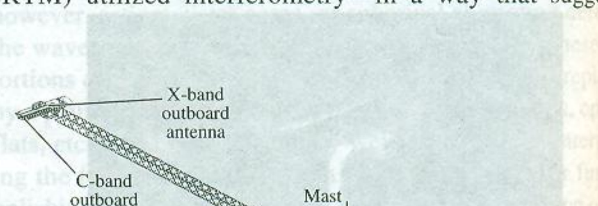

Figure 9.58 The Shuttle Endeavour carried the main C-band transmitter-receiver antenna in its cargo bay and a second receiver at the end of a 60-m long mast.