

A simple demonstration of frustrated total internal reflection

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We describe a quantitative and inexpensive method of demonstrating frustrated total internal reflection based on the attenuation of a laser beam through a wedge-shaped air gap between two glass prisms. An advantage of this method is that the thickness of the air gap can be determined by observing interference effects with only a slight modification of the setup. The experiment avoids the need to employ high-precision translation stages as is often the case in standard demonstrations of frustrated total internal reflection. © 2008 American Association of Physics Teachers.

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I. INTRODUCTION

Total internal reflection occurs at the boundary of two dielectric media if a light wave propagates from an optically denser medium (refractive index n_1) into one of lower density (refractive index n_2), and the angle of incidence exceeds the critical angle, given by $\arcsin(n_2/n_1)$. However, even if the angle of incidence is larger than the critical angle, the electric field penetrates into the adjacent medium in the form of an evanescent wave. The penetration depth is in the order of a wavelength, and can be measured by “frustrating” the reflection by adding an optically dense third medium at a distance on the order of a wavelength from the first medium, in which case the evanescent wave is coupled to the third medium and carries energy. The coupling strength falls off nearly exponentially as the distance between the first and third media is increased. Although many authors point out the analogy to quantum tunneling,^{1,2} frustrated total internal reflection (sometimes called optical tunneling) is an interesting phenomenon in its own right and demonstrates the wave nature of light in a unique fashion.

It is difficult to set up a quantitative experiment on frustrated total internal reflection that is suitable for an undergraduate laboratory course. The major obstacle is the difficulty of positioning two parallel flat glass surfaces sufficiently close to each other and controlling and measuring their distance precisely. Most experimental setups that have been tried employ an adjustable air gap between two right-angle prisms. If the exponential fall-off of the coupling is to be measured with good accuracy, we need a set-up that permits varying the thickness of a gap precisely while keeping the two surfaces parallel to each other. To counter this difficulty, many experiments have been done in the far infrared or microwave region.^{3,4}

In this paper we describe a conceptually and experimentally simple setup in the visible range of the spectrum. The experiment does not require precision measurements, but gives reasonable results for the attenuation coefficients. Our configuration is similar to that of Carlos.⁵ However, we use a laser instead of spectral lamps as the light source. The use of the laser simplifies the experiment and increases the accuracy of the measurement. A different approach to the problem has very recently been presented by You *et al.*⁶ and their experiments complement ours. Direct probing of evanescent optical waves has recently been described by Papanthassogloua.⁷

II. THEORETICAL BACKGROUND

Although the decay of the evanescent wave is exponential,⁸ if a frustrating medium is present, we have to account for multiple reflections on all surfaces. For a thorough treatment of the problem, the reader is referred to Ref. 1 or Ref. 9 for the transmitted power, and Ref. 10 for the various phase shifts involved. We only outline the procedure here. For the geometry and definition of angles, see Fig. 1 which shows two semi-infinite media surrounding a dielectric of thickness d . We will consider only parallel surfaces, because this case is the one relevant to our experiments. The refractive indices are n_0 , n_1 , and n_2 , respectively, and the angles of incidence are ϕ_0 , ϕ_1 , and ϕ_2 in the corresponding media.

We denote the complex reflection coefficients of the first and second surfaces by r_{01} and r_{12} for the forward directions and assume a lossless medium. The complex reflection coefficient of the thin dielectric layer can be obtained by summing all backward propagating waves as

$$r = \frac{r_{01} + r_{12}e^{-i\delta}}{1 + r_{12}r_{01}e^{-i\delta}}, \quad (1)$$

where the phase shift is defined as

$$\delta = \frac{4\pi d}{\lambda} \sqrt{n_1^2 - n_0^2 \sin^2 \phi_0}. \quad (2)$$

The reflection coefficients r_{01} and r_{12} can be calculated from the Fresnel equations for both polarizations of light. This result holds for any angle of incidence. When the condition for total internal reflection is satisfied, that is, when $n_0 \sin \phi_0 > n_1$, the phase shift is imaginary, resulting in a damped real field. If we substitute the Fresnel coefficients and the phase shift into Eq. (1) and express the transmitted intensity coefficient as $T = 1 - |r|^2$, we find

$$\frac{1}{1 - |r|^2} = \frac{1}{T} = \alpha \sinh^2(\beta d/2) + \gamma, \quad (3)$$

where

$$\beta = \frac{4\pi}{\lambda} \sqrt{n_0^2 \sin^2 \phi_0 - n_1^2}. \quad (4)$$

Depending on the polarization of the light relative to the plane of incidence, we have

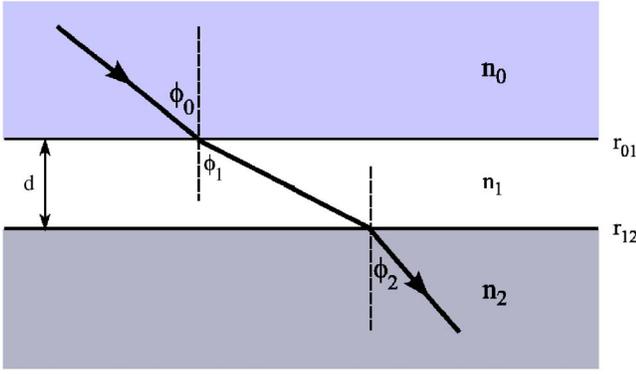


Fig. 1. Transmission of light through a plane parallel dielectric layer of thickness d and the definition of the angles.

$$\alpha_s = \frac{(n^2 - 1)(\tilde{n}^2 - 1)}{4n^2 \cos \phi_0 (\tilde{n}^2 \sin^2 \phi_0 - 1) \sqrt{\tilde{n}^2 - \sin^2 \phi_1}}, \quad (5)$$

$$\gamma_s = \frac{[\sqrt{\tilde{n}^2 - \sin^2 \phi_0} + \cos \phi_0]^2}{4 \cos \phi_0 \sqrt{\tilde{n}^2 - \sin^2 \phi_1}}, \quad (6)$$

$$\alpha_p = \alpha_s [(n^2 + 1) \sin \phi_0 - 1] / \tilde{n}^2, \quad (7)$$

$$\gamma_p = \frac{[\sqrt{\tilde{n}^2 - \sin^2 \phi_1} + \tilde{n}^2 \cos \phi_0]^2}{4\tilde{n}^2 \cos \phi_0 \sqrt{\tilde{n}^2 - \sin^2 \phi_1}}, \quad (8)$$

with $\tilde{n} = n_2/n_0$ and $n = n_0/n_1$. The subscripts s and p refer to the perpendicular and parallel polarization, respectively. For the special case $n_0 = n_2$, γ reduces to $\gamma_s = \gamma_p = 1$.

Note that for angles larger than the critical angle, $\alpha \sinh^2(\beta d/2)$ increases indefinitely as a function of d , while γ is constant. Therefore, for large d , γ can be neglected, and we obtain

$$T = \frac{1}{\alpha \sinh^2(\beta d/2) + \gamma} \approx \frac{1}{\alpha \sinh^2(\beta d/2)} \approx \frac{1}{\alpha} e^{-\beta d}. \quad (9)$$

We use the definition of β in Eq. (4) and consider Eqs. (5)–(7) and note that n , $n^2 - 1$, and $\cos \phi_0$ are of the order of 1. It is then easy to see that the exponential approximation can be made for $d \gtrsim \lambda/4$. We will be considering this limit, that is, when the coupling between the two high refractive index media is exponential. Also note that, although the absolute value of the transmission depends on the polarization through $\alpha_{s,p}$, its exponential attenuation factor β does not.

III. EXPERIMENT

In the standard approach to demonstrating and measuring frustrated total internal reflection, two prisms with parallel surfaces act as the semi-infinite media, with a plane-parallel small air gap between them. One of the prisms is then moved with respect to the other, and the transmitted power is recorded as a function of the separation. In our approach we move both prisms with respect to the laser while making sure that, as we move them, the laser beam scans through various separation distances. In practice, a small spacer is inserted, forming a wedge-shaped air gap. The advantage of this configuration is that the required mechanical precision can be

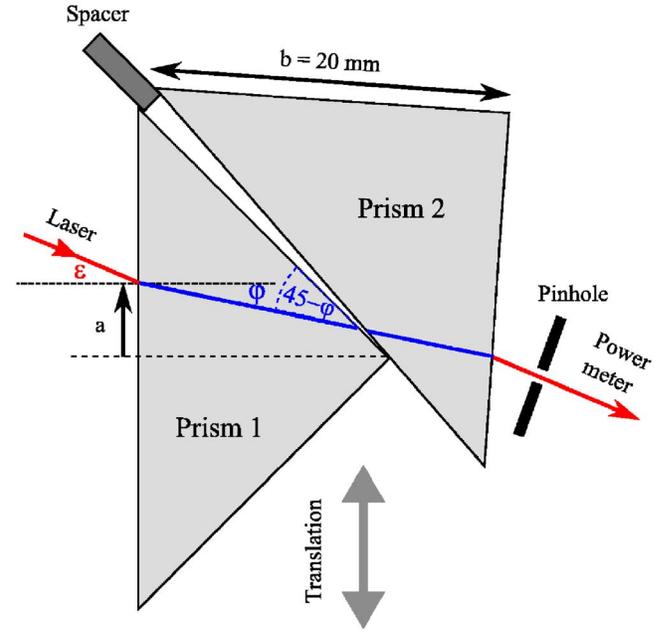


Fig. 2. Experimental setup. The size of the wedge is grossly exaggerated. In the picture ϵ is positive, given that the laser is entering from above the normal. The corresponding ϕ is also defined to be positive.

reduced considerably, and instead of having to measure distances on the order of a wavelength, it suffices to achieve a resolution of some tenths of a mm.

The experimental setup is shown in Fig. 2. Two right-angle prisms¹¹ are mounted next to each other on a rotation stage that is mounted on a translation stage. A small wedge is produced by inserting a thin aluminum foil of thickness t between one pair of the edges, while the apex of one prism serves as a pivot point. Then the two prisms are pressed against each other at the free apex and the opposing edge. Because we used foils that are about $10 \mu\text{m}$ thick, the angle between the two prisms is very small, so that, for the purpose of calculating the transmission of light, we can treat the surfaces as parallel.

A He-Ne (Ref. 12) laser is then positioned at around 20 cm from the prisms and is directed at a nearly zero angle of incidence at the free hypotenuse of prism 1. The transmitted intensity is measured by a power meter facing the laser.¹³ To exclude any stray or scattered light, a small pinhole is placed in front of the power meter.

As far as the underlying physics is concerned, the usual configuration (the two hypotenuses facing each other) should work as well. However, we found that these particular surfaces of our prisms performed better and had higher optical quality than the two hypotenuses.

By translating the prisms, the light encounters different thicknesses of the air gap and the transmitted intensity will change in accordance with Eq. (9). The thickness of the air gap can easily be calculated as shown in Fig. 2. For reference, we discuss the general case of angle of incidence ϵ with the convention that it is positive if the light is above the normal vector of the surface before reaching the first prism. We define the angle of refraction ϕ to be positive if the light propagates below the first normal.

According to the relation $n_{\text{air}} \sin \epsilon = n_{\text{pr}} \sin \phi$ ($n_{\text{air}} \approx 1$),

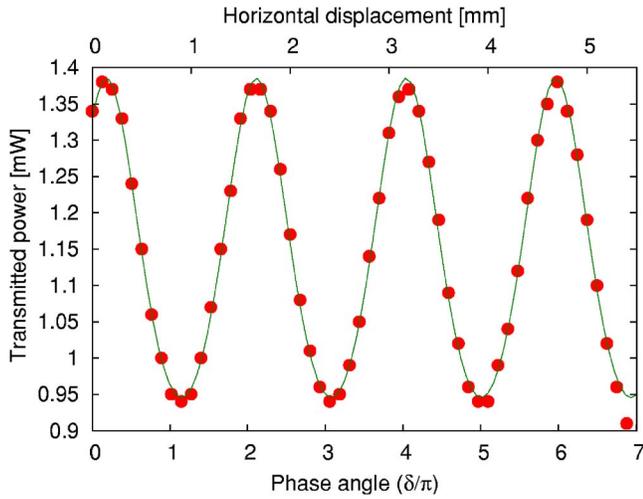


Fig. 3. Transmitted intensity in the Fabry–Perot configuration as a function of the horizontal displacement (top). The bottom scale shows the phase angle δ in units of π . The laser polarization is horizontal, $\varepsilon = -8^\circ \pm 1'$. The origin of the horizontal axis is arbitrary.

$$\varphi = \arcsin\left(\frac{\sin \varepsilon}{n_{\text{pr}}}\right) \approx \frac{\varepsilon}{n_{\text{pr}}}, \quad (10)$$

where the approximation holds for small angles. If we use this simplification and some elementary geometry, we arrive at the result that the laser light encounters an air gap of thickness

$$d(a, \varepsilon) = t \frac{a}{b\sqrt{2}} \left[1 + \tan\left(45^\circ + \frac{\varepsilon}{n_{\text{pr}}}\right) \right] = c(\varepsilon)a, \quad (11)$$

where a is the distance from the midpoint of the hypotenuse to the point of entry of the laser beam, b is the length of the cathetus, and t is the thickness of the foil.

The variation of the thickness of the air gap can easily be determined with the present setup, and this measurement provides an additional useful exercise for students. Furthermore, the foils might be crumpled, or the pressure on the two prisms might change over longer periods of time, leading to an erroneous slope $c(\varepsilon)$ in Eq. (11). Thus, it is better to measure the thickness of the air gap in a way that is independent of the mechanical parameters.

All we have to do is to turn the prisms in such a way that the condition for total internal reflection is not satisfied. In this case this setup becomes a Fabry–Perot etalon (with a very low finesse, because the reflectivity of the glass surfaces is only about 10% at an angle of incidence $\varepsilon = -8^\circ$) for which the separation of the two reflecting surfaces is given by Eq. (11). Hence, the modulation of the transmitted intensity will be only of the order of 20%, which is adequate to determine the lateral displacement needed to change the thickness of the air gap by one (or half) wavelength. A typical measurement is shown in Fig. 3. The angle of incidence in this case was $\varepsilon = -8^\circ \pm 1'$, which is slightly smaller than the ε corresponding to the critical angle, $\varepsilon_{\text{cr}} = -5^\circ 44'$. As we will see, a horizontal translation of about 1.5 mm corresponds to a phase shift of $\delta = 2\pi$. If we use Eq. (2) for the phase shift, with $\phi_0 = 45^\circ + \varepsilon/n_{\text{pr}}$, we arrive at the result that the separation of the two prisms changes by about 1.35 λ for a translation of 1 mm. It is important to realize that this measurement gives only $c(\varepsilon)$, and not the absolute separation. Because the

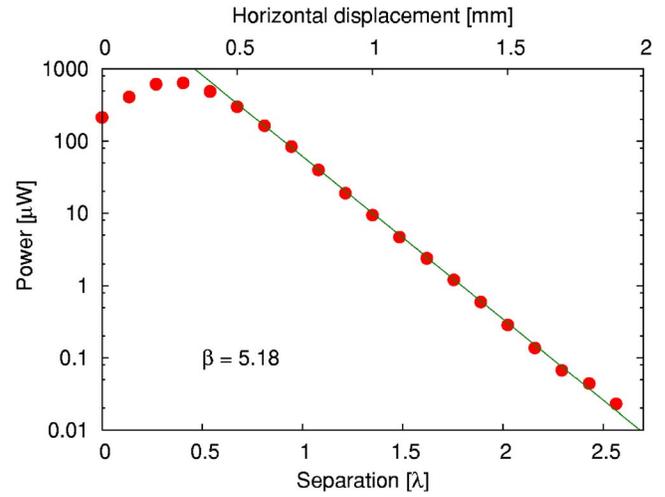


Fig. 4. Transmitted intensity in the frustrated total internal reflection configuration as a function of the horizontal displacement. The laser polarization is horizontal; that is, the light is polarized in the plane of incidence, and the angle of incidence is $\varepsilon = -24' \pm 1'$. The top scale shows the actual translation; on the bottom the prism separation is plotted in units of wavelength.

spacing of successive interference maxima is constant, the prisms do not suffer from any mechanical deformation over the measured range.

If the stage is set at an angle larger than $\varepsilon_{\text{cr}} = -5^\circ 44'$, we observe frustrated total internal reflection and its characteristic exponential decay as the prisms are translated, instead of periodic oscillations in the transmitted power. A typical plot of the transmitted power as a function of the horizontal displacement is shown in Fig. 4. The polarization was horizontal, that is, the electric field vector oscillated in the plane of incidence. The origin of the displacement scale is arbitrary, and is on the left-hand side of the apex of prism 1. The initial increase in power is due to the fact that the laser moves through the apex, where there is much light scattering, and where the left-hand side of the beam, which suffers total internal reflection on the free surface, does not contribute to the transmitted power. Initially the transmittance is expected to show a $\sinh^2(\beta d/2)$ dependence. However, after moving sufficiently far from the apex, the power shows an exponential decrease over a range of more than four orders of magnitude. This drastic change in the power can easily be discerned by the naked eye.

The fact that the laser beam has a finite width and, therefore, samples a range of gap thicknesses, does not alter the measured exponential decay constant, provided that the laser beam is much narrower than the range of its lateral displacement. To show that the finite width of the laser beam does not enter critically, we assume that the laser beam is far from the apex; that is, the attenuation is exponential in the distance of the two glass surfaces. Then the measured transmitted power at position a is given by

$$P_{\text{tr}}(a) = \int da' dy e^{-\beta c(\varepsilon)a'} P(a', y), \quad (12)$$

where $P(a', y)$ is the power density of the laser on the first surface. Here we also assume that the attenuation does not depend on the vertical coordinate, y .

If we move the laser by a distance ℓ , the transmitted power becomes

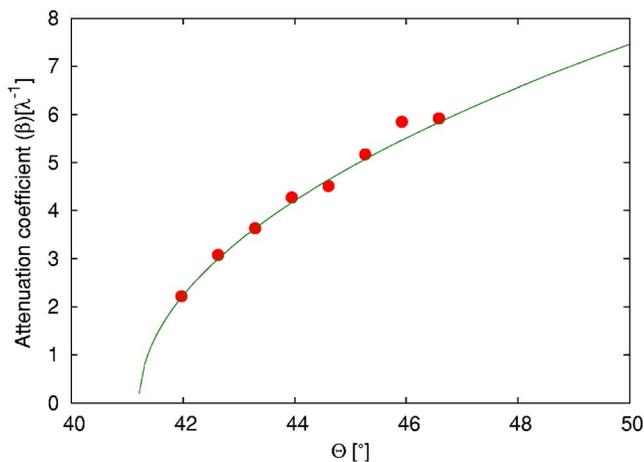


Fig. 5. Measured attenuation coefficient β for a glass-air interface as a function of the angle of incidence at the second glass-air interface, $\Theta = 45^\circ + \varphi$. Also shown is the theoretical dependence given by Eq. (4). The accuracy of the angle measurements is $1'$.

$$P_{\text{tr}}(a + \ell) = \iint da' dy e^{-\beta c(\varepsilon)a'} P(a' - \ell, y) \quad (13a)$$

$$= e^{-\beta c(\varepsilon)\ell} \iint da' dy e^{-\beta c(\varepsilon)a'} P(a', y) \\ = e^{-\beta c(\varepsilon)\ell} P_{\text{tr}}(a). \quad (13b)$$

Thus, the ratio of the new intensity to the old one is simply $e^{\beta c(\varepsilon)\ell}$, independent of the laser profile. In this derivation we made use of the fact that the transmittance is exponential in the distance of the prisms. Thus, this result does not hold when the beam is so large or so close to the apex that $\sinh^2(\beta d/2)$ cannot be replaced by a single exponential. Therefore, a narrow laser beam is preferable if we wish to obtain an exponential decay over a significant range of intensities. For this reason, the laser beam should not be placed too far away from the prisms.

The exponential decay constant β can be deduced from graphs such as Fig. 4. We measured β as a function of the

angle of incidence at the second glass-air interface, $45^\circ + \varphi$, and compared it to theory, as is shown in Fig. 5. The accuracy of the angle measurement is $1'$. The agreement between theory and the observed dependence on the angle of incidence is satisfactory.

IV. CONCLUSION

We have presented a simple experimental configuration for the demonstration of frustrated total internal reflection. We also showed that this setup can be used for quantitative measurements and gives good agreement with theory. Neither the concept nor the design are beyond what can be realized in an undergraduate laboratory. However, technical improvements in mounting the prisms may be advisable. As an extension of the experiments, the effect can be studied for both polarizations of light.

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¹¹Edmund Optics, PN: NT32-547, refractive index $n=1.518$ at $\lambda=632.8$ nm.

¹²2.2 mW power from JDS Uniphase, (www.jdsu.com/), Model No. 1101P.

¹³Coherent FieldMaster with an LM-2 VIS head, (www.coherent.com/). The sensitivity range is 10 nW–50 mW.

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